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Applications of Matrix Multiplication

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ABSTRACT

In this paper we present some interesting applications of the matrix's multiplication, that include the Leslie matrix and population change, which we calculate this kind of changes from year to other year by matrix multiplication. Another important part of the paper is Analysis of Traffic Flow, we represent the flow of traffic through a network of one-way streets. Another much important part is the production costs, this is fantastic usage of matrix multiplication, in which, A company manufactures three products. Its production expenses are divided into three categories, here in this paper we well describe this beautiful issue. By matrix multiplication, we can encode and decode messages. To encode a message, we choose an $n \times n$ invertible matrix A and multiply the uncoded row matrices (on the right) by A to obtain coded row matrices, this idea will be clarify by some useful examples. Also we used to study certain relationships between objects by matrix multiplication. We will clarify all of these applications by useful examples. In this paper, we present six different applications of matrix multiplication.

Keywords- Matrix multiplication, matrices, population Change, population, traffic flow.

I. INTRODUCTION

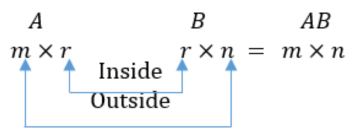
This required to define the product of two matrices, and also has need to describe matrix multiplication. This article aims to promote several application aspects of matrix multiplication [1]. Experience has led mathematicians to the following more useful definition of matrix multiplication.

Definition. The product of two matrices $A \in M_{m,n}(F)$ and $B \in M_{n,p}(F)$ (such that the number of columns n of A equals the number of rows n of B) is the unique matrix $AB \in M_{m,p}(F)$ such that

$$A(BX) = (AB)X$$

for all $X \in F^p$ [10].

Way to determine whether a product of two matrices is defined is to write down the size of the first factor and, to the right of it, write down the size of the second factor. As shows in the following, the inside numbers are the same, then the product is defined. The outside numbers then give the size of the product [3], we have



For example,

$$A = \begin{bmatrix} 5 & -1 & 4 \\ -3 & 6 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} 9 & 4 & -8 & 2 \\ 7 & 6 & -1 & 0 \\ -2 & 5 & 3 & -4 \end{bmatrix}.$$

Since A is a 2×3 matrix and B is a 3×4 matrix, the number of columns of A equals the number of rows of B (three in each case). Therefore, A and B can be multiplied, and the product matrix $C = AB$ is a 2×4 matrix, because A has two rows and B has four columns. To calculate each entry of C, we take the dot product of

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the appropriate row of A with the appropriate column of B. For example, to find c_{11}, we take the dot product of the 1st row of A with the 1st column of B:

$$c_{11} = [5, -1, 4] \cdot \begin{bmatrix} 9 \\ 7 \\ -2 \end{bmatrix} = (5)(9) + (-1)(7) + (4)(-2) = 45 - 7 - 8 = 30.$$

Find c_{23}, we take the dot product of the 2nd row of A with the 3rd column of B:

$$c_{23} = [-3, 6, 0] \cdot \begin{bmatrix} -8 \\ -1 \\ 3 \end{bmatrix} = (-3)(-8) + (6)(-1) + (0)(3) = 24 - 6 + 0 = 18.$$

The other entries are computed similarly, yielding

$$C = AB = \begin{bmatrix} 30 & 34 & -27 & -6 \\ 15 & 24 & 18 & -6 \end{bmatrix} [8].$$

Finally we found the product of tow matrices A and B in the form $\begin{bmatrix} 30 & 34 & -27 & -6 \\ 15 & 24 & 18 & -6 \end{bmatrix}$.

Note that, the matrix multiplication is not commutative: Usually $AB \neq BA$. But matrix multiplication is associative: $(AB)C = A(BC)$ and matrix operations are distributive: $A(B + C) = AB + AC$ and $(B + C)D = BD + CD$ [4]. The componentwise definition of scalar multiplication will come as no surprise. If A is an $m \times n$ matrix and c is a scalar, then the scalar multiple cA is the $m \times n$ matrix obtained by multiplying each entry of A by c. More formally, we have

$$cA = c[a_{ij}] = [ca_{ij}] \text{ [3].}$$

Now we will discuss the main part of the paper, applications of matrix multiplication in population Change, population movement, analysis of traffic flow, $(0, 1)$-matrices, production cost and encode and decode messages by matrix multiplication.

II. THE LESLIE MATRIX AND POPULATION CHANGE

The population of a colony of animals depends on the birth and mortality rates for the various age groups of the colony. For example, suppose that the members of a colony of mammals have a life span of less than 3 years. To study the birth rates of the colony, we divide the females into three age groups: those with ages less than 1, those with ages between 1 and 2, and those of age 2. From the mortality rates of the colony, we know that 40% of newborn females survive to age 1 and that 50% of females of age 1 survive to age 2. We need to observe only the rates at which females in each age group give birth to female offspring since there is usually a known relationship between the number of male and female offspring in the colony. Suppose that the females under 1 year of age do not give birth; those with x_d Cages between 1 and 2 have, on average, two female offspring; and those of age 2 have, on average, one female offspring. Let x_1, x_2, and x_3 be the numbers of females in the first, second, and third age groups, respectively, at the present time, and let y_1, $y_2$and y_3 be the numbers of females in the corresponding groups for the next year. The changes from this year to next year are depicted in Table1.

Table 1

Age in years	Current year	Next year
$0-1$	x_1	y_1
$1-2$	x_2	y_2
$2-3$	x_3	y_3

The vector $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ is the population distribution for the female population of the colony in the present year. We can use the preceding information to predict the population distribution for the following year, which is given by the vector $y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$.

Note that y_1, the number of females under age 1 in next year's population, is simply equal to the number of female offspring born during the current year. Since there are currently x_2 females of age $1-2$, each of which has, on average, 2 female offspring, and x_3 females of age $2-3$, each of which has, on average, 1 female offspring, we have the following formula for y_1:

$$y_1 = 2x_2 + x_3$$

The number y_2 is the total number of females in the second age group for next year. Because these females are in the first age group this year, and because only 40% of them will survive to the next year, we have that $y_2 = 0.4x_1$. Similarly, $y_3 = 0.5x_2$. Collecting these three equations, we have

$$\begin{aligned} y_1 &= 2.0x_2 + 1.0x_3 \\ y_2 &= 0.4x_1 \\ y_3 &= 0.5x_2. \end{aligned}$$

These three equations can be represented by the single matrix equation $y = Ax$, where x and y are the population distributions as previously defined and A is the 3×3 matrix

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$$A = \begin{bmatrix} 0.0 & 2.0 & 1.0 \\ 0.4 & 0.0 & 0.0 \\ 0.0 & 0.5 & 0.0 \end{bmatrix}.$$

For example, suppose that $x = \begin{bmatrix} 1000 \\ 1000 \\ 1000 \end{bmatrix}$; that is, there are currently 1000 females in each age group. Then

$$y = Ax = \begin{bmatrix} 0.0 & 2.0 & 1.0 \\ 0.4 & 0.0 & 0.0 \\ 0.0 & 0.5 & 0.0 \end{bmatrix} \begin{bmatrix} 1000 \\ 1000 \\ 1000 \end{bmatrix} = \begin{bmatrix} 3000 \\ 400 \\ 500 \end{bmatrix}.$$

So one year later there are 3000 females under 1 year of age, 400 females who are between 1 and 2 years old, and 500 females who are 2 years old.

For each positive integer k, let p_k denote the population distribution k years after a given initial population distribution p_0. In the preceding example,

$$p_0 = x = \begin{bmatrix} 1000 \\ 1000 \\ 1000 \end{bmatrix} \text{ and } \quad p_1 = y = \begin{bmatrix} 3000 \\ 400 \\ 500 \end{bmatrix}.$$

Then, for any positive integer k, we have that $p_k = Ap_{k-1}$. Thus $p_k = Ap_{k-1} = A^2 p_{k-2} = \cdots = A^k p_0$.

In this way, we may predict population trends over the long term. For example, to predict the population distribution after 10 years, we compute $p_{10} = A^{10} p_0$. Thus

$$p_{10} = A^{10} p_0 = \begin{bmatrix} 1987 \\ 851 \\ 387 \end{bmatrix}.$$

where each entry is rounded off to the nearest whole number. If we continue this process in increments of 10 years, we find that (rounding to whole numbers)

$$p_{20} = \begin{bmatrix} 2043 \\ 819 \\ 408 \end{bmatrix} \quad \text{and} \quad p_{30} = p_{40} = \begin{bmatrix} 2045 \\ 818 \\ 409 \end{bmatrix}.$$

It appears that the population stabilizes after 30 years. In fact, for the vector

$$z = \begin{pmatrix} 2045 \\ 818 \\ 409 \end{pmatrix}$$

we have that $Az = z$ precisely. Under this circumstance, the population distribution z is stable; that is, it does not change from year to year.

In general, whether or not the distribution of an animal population stabilizes for
a colony depends on the survival and birth rates of the age groups.

We can generalize this situation to an arbitrary colony of animals. Suppose that we divide the females of the colony into n age groups, where x_i is the number of members in the $i\ th$ group. The duration of time in an individual age group need not be a year, but the various durations should be equal. Let $x = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$ be the population distribution of the females of the colony, p_i be the portion of females in the $i\ th$ group who survive to the $(i+1)st$ group, and $b_i$be the average number of female offspring of a member of the $i\ th$ age group. If $y = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix}$ is the population for the next time period, then

$$\begin{aligned} y_1 &= b_1 x_1 + \ b_2 x_2 + \cdots + b_n x_n \\ y_2 &= p_1 x_1 \\ &\vdots \\ y_3 &= \qquad p_2 x_2. \\ y_n &= \\ p_{n-1} x_{n-1}. \end{aligned}$$

Therefore, for

$$A = \begin{pmatrix} b_1 & b_2 & \cdots & b_n \\ p_1 & 0 & \cdots & 0 \\ 0 & p_2 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots \ p_{n-1} & 0 \end{pmatrix},$$

We have

$$y = Ax.$$

The matrix A is called the Leslie matrix for the population. The name is due to
P. H. Leslie, who introduced this matrix in the 1940s. So if x_0 is the initial population distribution, then the distribution after k time intervals is $x_k = A^k x_0$ [6]. We continue this discussion as movement of population.

III. POPULATION MOVEMENT

A subject of interest to demographers is the movement of- populations or groups of people from one region to another. We consider here a simple model of the changes in the population of a certain city and its surrounding suburbs **over** a period of years.

Fix an initial year, say 1990, and denote the populations of the city and suburbs that year by r_0 and s_0**,** respectively. Let x_0**,** be the population vector

$$x_0 = \begin{pmatrix} r_0 \\ s_0 \end{pmatrix} \begin{matrix} \text{City population. I990} \\ \text{subueban popuirtain 1990} \end{matrix}$$

For 1991 and subsequent years, denote the population of the city and suburbs **by** the vectors

$$x_1 = \begin{pmatrix} r_1 \\ s_1 \end{pmatrix}, \quad x_2 = \begin{pmatrix} r_2 \\ s_2 \end{pmatrix}, \quad x_3 = \begin{pmatrix} r_3 \\ s_3 \end{pmatrix}, \ldots$$

Our goal is to describe mathematically how these vectors might be related. Suppose demographic studies show that each year about 5% of the city's population moves to the suburbs (and 95% remain in the city), while 3% of the suburban population moves to the city (and 97% remain in the suburbs). See figure 1.

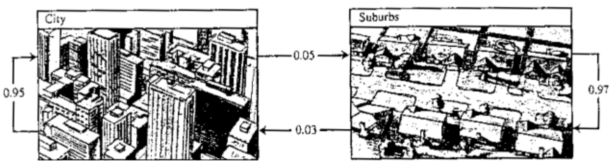

Figure 1. Annual percentage migration between city and suburbs.

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After one year, the original r_0, persons in the city are now distributed between city and suburbs as

$$\begin{pmatrix}0.95r_0\\0.05r_0\end{pmatrix}$$

$$= r_0\begin{pmatrix}0.95\\0.05\end{pmatrix}\begin{matrix}\text{Move to city}\\\text{Remain in suburbs}\end{matrix} \qquad (1)$$

The s_0 persons in the suburbs in 1990 are distributed one year later as

$$s_0\begin{pmatrix}0.05\\0.97\end{pmatrix}\begin{matrix}\text{Move to city}\\\text{Remain in suburbs}\end{matrix} \qquad (2)$$

The vectors in (1) and (2) account for all of the population in 1991, Thus

$$\begin{pmatrix}r_1\\s_1\end{pmatrix} = r_0\begin{pmatrix}0.95\\0.05\end{pmatrix} + s_0\begin{pmatrix}0.05\\0.97\end{pmatrix} = \begin{pmatrix}0.95 & 0.03\\0.05 & 0.97\end{pmatrix}\begin{pmatrix}r_0\\s_0\end{pmatrix}$$

That is,

$$x_1 = Mx_0 \qquad (3)$$

where M is the migration matrix determined by the following table:

$$\begin{matrix} & \text{Form} & \\ \text{City} & \text{Suburbs} & \text{To}\end{matrix}$$

$$\begin{pmatrix}0.95 & 0.03\\0.05 & 0.97\end{pmatrix}\begin{matrix}\text{city}\\\text{suburbs}\end{matrix}$$

Equation (4) describes how the population changes from 1990 to 199 1. If the migration percentages remain constant, then the change from 1991 to 1992 is given by

$$x_2 = Mx_1$$

and similarly for 1992 to I993 and subsequent years. In general

$$x_{k+1} = Mx_k \quad \text{for } k = 0, 1, 2, \ldots \qquad (4)$$

The sequence of vectors $\{x_0, x_1, x_2, \ldots\}$ describes the population of the city/suburban region over a period of years, and the change in the population from one year to the next is given by (4). Here we have an example.

Example 1. Compute the population of the region just described for the years 1991 and 1992, given that the population in 1990 was 600,000 in the city and 300,000 in the suburbs.

Solution: The initial population in 1990 is $x_0 = \begin{pmatrix}600000\\400000\end{pmatrix}$. For 1991,

$$x_1 = \begin{pmatrix}0.95 & 0.03\\0.05 & 0.97\end{pmatrix}\begin{pmatrix}600000\\400000\end{pmatrix} = \begin{pmatrix}582000\\418000\end{pmatrix}.$$

For 1992,

$$x_2 = Mx_1 = \begin{pmatrix}0.95 & 0.03\\0.05 & 0.97\end{pmatrix}\begin{pmatrix}582000\\418000\end{pmatrix} = \begin{pmatrix}565440\\434560\end{pmatrix}$$ [2].

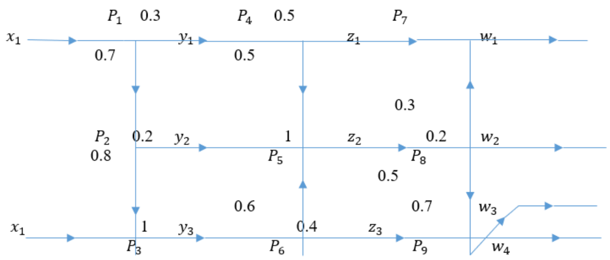

Figure 2. Traffic flow along one-way streets

IV. ANALYSIS OF TRAFFIC FLOW

Figure 1 represents the flow of traffic through a network of one-way streets, with arrows indicating the direction of traffic flow. The number on any street beyond an intersection is the portion of the traffic entering the street from that intersection. For example, 30% of the traffic leaving intersection P_1 goes to P_4, and the other 70% goes to P_2. Notice that all the traffic leaving P_5 goes to P_8.

Suppose that on a particular day, x_1 cars enter the network from the left of P_1, and x_2 cars enter from the left of P_3. Let w_1, w_2, w_3, and w_4 represent the number of cars leaving the network along the exits to the right. We wish to determine the values of the w_i's. At first glance, this problem seems overwhelming since there are so many routes for the traffic. However, if we decompose the problem into several simpler ones, we can first solve the simpler ones individually and then combine their solutions to obtain the values of the w_i's.

We begin with only the portion of the network involving intersections P_1, P_2, and P_3. Let y_1, y_2, and y_3 each be the number of cars that exit along each of the three eastward routes, respectively. To find y_1, notice that 30% of all cars entering P_1 continue on to P_4. Therefore, $y_1 = 0.30x_1 = 0.30x1$. Also, $0.7x_1$ of the cars turn right at P_1, and of these, 20% turn left at P_2. Because these are the only cars to do so, it follows that $y_2 = (0.2)(0.7)x_1 = 0.14x_1$. Furthermore, since 80% of the cars entering P_2 continue on to P_3, the number of such cars is $(0.8)(0.7)x_1 = 0.56x_1$. Finally, all the cars entering P_3 from the left use the street between P_3 and P_6, so $y_3 = 0.56x_1 + x_2$. Summarizing, we have

$$\begin{aligned}y_1 &= 0.30x_1\\y_2 &= 0.14x_1\\y_3 &= 0.56x_1 + x_2.\end{aligned}$$

We can express this system of equations by the single matrix equation $y = Ax$, where

$$y = \begin{pmatrix}y_1\\y_2\\y_3\end{pmatrix} \quad A = \begin{pmatrix}0.30 & 0\\0.14 & 0\\0.56 & 1\end{pmatrix} \quad \text{and} \quad x = \begin{pmatrix}x_1\\x_2\end{pmatrix}.$$

Now consider the next set of intersections P_4, P_5, and P_6. If we let z_1, z_2, and z_3 represent the numbers of cars that exit from the right of P_4, P_5, and P_6, respectively, then by a similar analysis, we have

$$\begin{aligned}z_1 &= 0.5y_1\\z_2 &= 0.5y_1 + y_2 + 0.6y_3\\z_3 &= \ 0.5y_3\end{aligned}$$

or $z = By$, where

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$$z = \begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 0.5 & 0 & 0 \\ 0.5 & 1 & 0.6 \\ 0 & 0 & 0.4 \end{pmatrix}.$$

Finally, if we set

$$w = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{pmatrix} \quad \text{and} \quad C = \begin{pmatrix} 1 & 0.30 & 0 \\ 0 & 0.20 & 0 \\ 0 & 0.35 & 0.7 \\ 0 & 0.15 & 0.3 \end{pmatrix}$$

then by a similar argument, we have $w = Cz$. It follows that

$$w = Cz = C(By) = (CB)Ax = (CBA)x.$$

Let $M = CBA$. Then

$$M = \begin{pmatrix} 1 & 0.30 & 0 \\ 0 & 0.20 & 0 \\ 0 & 0.35 & 0.7 \\ 0 & 0.15 & 0.3 \end{pmatrix} \begin{pmatrix} 0.5 & 0 & 0 \\ 0.5 & 1 & 0.6 \\ 0 & 0 & 0.4 \end{pmatrix} \begin{pmatrix} 0.30 & 0 \\ 0.14 & 0 \\ 0.56 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0.3378 & 0.18 \\ 0.1252 & 0.12 \\ 0.3759 & 0.49 \\ 0.1611 & 0.21 \end{pmatrix}.$$

For example, if 1000 cars enter the traffic pattern at P_1 and 2000 enter at P_3, then, for $x = \begin{pmatrix} 1000 \\ 2000 \end{pmatrix}$, we have

$$w = Mx = \begin{pmatrix} 0.3378 & 0.18 \\ 0.1252 & 0.12 \\ 0.3759 & 0.49 \\ 0.1611 & 0.21 \end{pmatrix} \begin{pmatrix} 1000 \\ 2000 \end{pmatrix}$$

$$= \begin{pmatrix} 697.8 \\ 365.2 \\ 1355.9 \\ 581.1 \end{pmatrix}.$$

Naturally, the actual number of cars traveling on any path is a whole number, unlike the entries of w. Since these calculations are based on percentages, we cannot expect the answers to be exact. For example, approximately 698 cars exit the traffic pattern at P_7, and 365 cars exit the pattern at P_8.

We can apply the same analysis if the quantities studied represent *rates* of traffic flow—for example, the number of cars per hour—rather than the total number of cars [6].

V. Production Costs

A company manufactures three products. Its production expenses are divided into three categories. In each category, an estimate is given for the cost of producing a single item of each product. An estimate is also made of the amount of each product to be produced per quarter. These estimates are given in Tables 2 and 3. At its stockholders' meeting, the company would like to present a single table showing the total costs for each quarter in each of the three categories: raw materials, labor, and overhead.

Table 2: Production Costs per Item (dollars)

Expenses	Product		
	A	B	C
Raw materials	0.10	0.30	0.15
Labor	0.30	0.40	0.25
Overhead and miscellaneous	0.10	0.20	0.15

Table 3: Amount Produced per Quarter

Product	Product			
	Summer	Fall	Winter	Spring
A	4000	4500	4500	4000
B	2000	2600	2400	2200
C	5800	6200	6000	6000

Solution. Let us consider the problem in terms of matrices. Each of the two tables can be represented by a matrix, namely,

$$M = \begin{pmatrix} 0.10 & 0.30 & 0.15 \\ 0.30 & 0.40 & 0.25 \\ 0.10 & 0.20 & 0.15 \end{pmatrix}$$

And

$$P = \begin{pmatrix} 4000 & 4500 & 4500 & 4000 \\ 2000 & 2600 & 2400 & 2200 \\ 5800 & 6200 & 6000 & 6000 \end{pmatrix}$$

If we form the product MP, the first column of MP will represent the costs for the summer quarter:

Raw materials: $(0.10)(4000) + (0.30)(2000) + (0.15)(5800) = 1870$

Labor: $(0.30)(4000) + (0.40)(2000) + (0.25)(5800) = 3450$

Overhead and miscellaneous: $(0.10)(4000) + (0.20)(2000) + (0.15)(5800) = 1670$

The costs for the fall quarter are given in the second column of MP:

Raw materials: $(0.10)(4500) + (0.30)(2600) + (0.15)(6200) = 2160$

Labor: $(0.30)(4500) + (0.40)(2600) + (0.25)(6200) = 3940$

Overhead and miscellaneous: $(0.10)(4500) + (0.20)(2600) + (0.15)(6200) = 1900$

Columns 3 and 4 of MP represent the costs for the winter and spring quarters, respectively. Thus, we have $MP = \begin{pmatrix} 1870 & 2160 & 2070 & 1960 \\ 3450 & 3940 & 3810 & 3580 \\ 1670 & 1900 & 1830 & 1740 \end{pmatrix}$

The entries in row 1 of MP represent the total cost of raw materials for each of the four quarters. The entries in rows 2 and 3 represent the total cost for labor and overhead, respectively, for each of the four quarters. The yearly expenses in each category may be obtained by adding the entries in each row. The numbers in each of the columns may be added to obtain the total

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production costs for each quarter. Table 4 summarizes the total production costs [9].

Table 4

Expenses	Product				
	Summer	Fall	winter	Spring	Year
Raw materials	1870	2160	2070	1960	8060
Labor	3450	3940	3810	3580	14780
Overhead and miscellaneous	1670	1900	1830	1740	7140
Total production cost	6990	8000	7710	7280	29980

VI. $(\mathbf{0,1})$ −MATRICES

Matrices can be used to study certain relationships between objects. For example, suppose that there are five countries, each of which maintains diplomatic relations with some of the others. To organize these relationships, we use a 5×5 matrix A defined as follows. For $1 \le i \le 5$, we let $a_{ii} = 0$, and for $i \ne j$,

$$a_{ii} = \begin{cases} 1 & \text{if country } i \text{ maintains diplomatic relations with country } j \\ 0 & \text{otherwise.} \end{cases}$$

Note that all the entries of A are zeros and ones. Matrices whose only entries are zeros and ones are called $(\mathbf{0}, \mathbf{1})$**-matrices**, and they are worthy of study in their own right. For purposes of illustration, suppose that

$$A = \begin{pmatrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{pmatrix}.$$

In this case, $A = AT$; that is, A is symmetric. The symmetry occurs here because the underlying relationship is symmetric. (That is, if country i maintains diplomatic relations with country j, then also country j maintains diplomatic relations with country i.) Such symmetry is true of many relationships of interest. Figure 3 gives us a visual guide to the relationship, where country i is shown to maintain diplomatic relations with country j if the dots representing the two countries are joined by a line segment. (The diagram in Figure 3 is called an *undirected graph*.)

Let us consider the significance of an entry of the matrix $B = A^2$; for example,

$$b_{23} = a_{21}a_{13} + a_{22}a_{23} + a_{23}a_{33} + a_{24}a_{43} + a_{25}a_{53}.$$

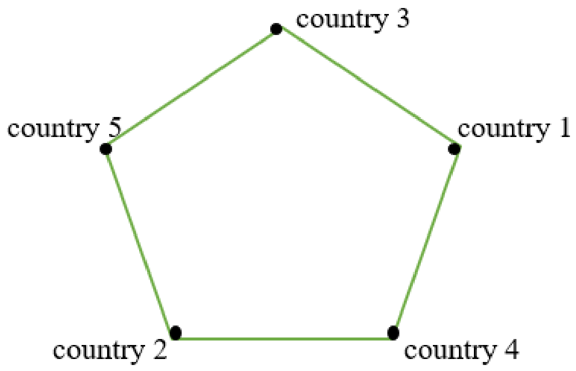

Figure 3. Diplomatic relations among countries

A typical term on the right-side of the equation has the form $a_{2k}a_{k3}$. This term is 1 if and only if both factors are 1-that is, if and only if country 2 maintains diplomatic relations with country k and country k maintains diplomatic relations with country 3. Thus b_{23} gives the number of countries that *link* country 2 and country 3. To see all of these entries, we compute

$$B = A^2 = \begin{pmatrix} 2 & 1 & 0 & 0 & 1 \\ 1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 1 & 2 & 1 \\ 1 & 0 & 0 & 1 & 2 \end{pmatrix}.$$

Since $b_{23} = 1$, there is exactly one country that links countries 2 and 3. A careful examination of the entries of A reveals that $a_{25} = a_{53} = 1$, and hence it is country 5 that serves as the link. (Other deductions are left for the exercises.) We can visualize the (i, j)-entry of A^2 by counting the number of ways to go from country i to country j in Figure 2 that use two line segments.

By looking at other powers of A, additional information may be obtained. For example, it can be shown that if A is an $n \times n$ (0, 1)-matrix and the (i, j)-entry of $A + A^2 + \cdots + A^{n-1}$ is nonzero, then there is a sequence of countries beginning with country i, ending with country j , and such that every pair of consecutive countries in the sequence maintains diplomatic relations. By means of such a sequence, countries i and j can communicate by passing a message only between countries that maintain diplomatic relations. Conversely, if the (i, j)-entry of $A + A^2 + \cdots + A^{n-1}$ is zero, then such communication between countries i and j is impossible.

Example 2. Consider a set of three countries, such that country 3 maintains diplomatic relations with both countries 1 and 2, and countries 1 and 2 do not maintain diplomatic relations with each other. These relationships can be described by the 3×3 $(0, 1)$-matrix

$$A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}.$$

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In this case, we have $A + A^2 = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 2 \end{pmatrix}$, and so countries 1 and 2 can communicate, even though they do not have diplomatic relations. Here the sequence linking them consists of countries $1, 3$ and 2.

A $(0, 1)$-matrix can also be used to resolve problems involving scheduling. Suppose, for example, that the administration of a small college with m students wants to plan the times for its n courses. The goal of such planning is to avoid scheduling popular courses at the same time. To minimize the number of time conflicts, the students are surveyed. Each student is asked which courses he or she would like to take during the following semester. The results of this survey may be put in matrix form.

Define the $m \times n$ matrix A as follows:

$$a_{ii} = \begin{cases} 1 \text{ if student } i \text{ wants to take course } j \\ 0 \text{ otherwise.} \end{cases}$$

In this case, the matrix product A^TA provides important information regarding the scheduling of course times. We begin with an interpretation of the entries of this matrix. Let $B = A^T$ and $C = A^TA = BA$. Then, for example,

$$c_{12} = b_{11}a_{12} + b_{12}a_{22} + \cdots + b_{1k}a_{k2} + \cdots + b_{1m}a_{m2} = a_{11}a_{12} + a_{21}a_{22} + \cdots + a_{k1}a_{k2} + \cdots + a_{m1}a_{m2}.$$

A typical term on the right side of the equation has the form $a_{k1}a_{k2}$. Now, $a_{k1}a_{k2} = 1$ if and only if $a_{k1} = 1$and $a_{k2} = 1$; that is, student k wants to take course 1 and course 2. So c_{12} represents the number of students who want to take both courses 1 and 2. In general, for $i \neq j$, c_{ij} is the number of students who want to take both course i and course . In addition, c_{ij} represents the number of students who desire class i. For more clarity we have another example as follows:

Example 3. Suppose that we have a group of 10 students and five courses. The results of the survey concerning course preferences are as follows:

Course Number

Student	1	2	3	4	5
1	1	0	1	0	1
2	0	0	0	1	1
3	1	0	0	0	0
4	0	1	1	0	1
5	0	0	0	0	0
6	1	1	0	0	0
7	0	0	1	0	1
8	0	1	0	1	0
9	1	0	1	0	1
10	0	0	0	1	0

Let A be the 10×5 matrix with entries from the previous table. Then

$$A^TA = \begin{pmatrix} 4 & 1 & 2 & 0 & 2 \\ 1 & 3 & 1 & 1 & 1 \\ 2 & 1 & 4 & 0 & 4 \\ 0 & 1 & 0 & 3 & 1 \\ 2 & 1 & 4 & 1 & 5 \end{pmatrix}.$$

From this matrix, we see that there are four students who want both course 3 and course 5. All other pairs of courses are wanted by at most two students. Furthermore, we see that four students want course 1, three students desire course 2, and so on. Thus, the trace of A^TA equals the total demand for these five courses (counting students as often as the number of courses they wish to take) if the courses are offered at different times.

Notice that although A is not symmetric, the matrix A^TA is symmetric. Hence we may save computational effort by computing only one of the (i, j)- and (i, j)-entries. As a final comment, it should be pointed out that many of these facts about $(0, 1)$-matrices can be adapted to apply to nonsymmetrical relationships [6]. Another important application of matrix multiplication is encoding and decoding messages, which will be describe as follows.

VII. ENCODE AND DECODE MESSAGES BY MATRIX MULTIPLICATION

Here we have fantastic use of matrix multiplication to encode and decode messages. To encode a message, choose an $n \times n$ invertible matrix A and multiply the uncoded row matrices (on the right) by A to obtain coded row matrices. Example 4 demonstrates this process.

Example 4. For Encoding a Message. We use the following invertible matrix $A = \begin{pmatrix} 1 & -2 & 2 \\ -1 & 1 & 3 \\ 1 & -1 & -4 \end{pmatrix}$ to encode the message MEET ME MONDAY.

The coded row matrices by multiplying each of the uncoded row matrices we find that by the matrix, as follows:

[13	5	5]	[20	0	13]	[5	0	13]	[15	14	4]	[1	25	0]
M	E	E	T	__	M	E	__	M	O	N	D	A	Y	___

Note the use of a blank space to fill out the last uncoded row matrix.

$$[13 \quad 5 \quad 5]\begin{pmatrix} 1 & -2 & 2 \\ -1 & 1 & 3 \\ 1 & -1 & -4 \end{pmatrix} = [13 \quad -26 \quad 21]$$

$$[20 \quad 0 \quad 13]\begin{pmatrix} 1 & -2 & 2 \\ -1 & 1 & 3 \\ 1 & -1 & -4 \end{pmatrix} = [33 \quad -53 \quad -12]$$

$$[5 \quad 0 \quad 13]\begin{pmatrix} 1 & -2 & 2 \\ -1 & 1 & 3 \\ 1 & -1 & -4 \end{pmatrix} = [18 \quad -23 \quad -42]$$

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$$[15 \quad 14 \quad 4]\begin{pmatrix} 1 & -2 & 2 \\ -1 & 1 & 3 \\ 1 & -1 & -4 \end{pmatrix} = [5 \quad -20 \quad 56]$$

$$[1 \quad 25 \quad 0]\begin{pmatrix} 1 & -2 & 2 \\ -1 & 1 & 3 \\ 1 & -1 & -4 \end{pmatrix} = [-24 \quad 23 \quad 77]$$

The sequence of coded row matrices is

$$[13 \ -26 \quad 21][33 \ -53 \ -12][18 \ -23 \ -42][5 \ -20 \quad 56][-24 \quad 23 \quad 77].$$

Finally, removing the matrix notation produces the following cryptogram.

$$13 \ -26 \ 21 \ 33 \ -53 \ -12 \ 18 \ -23 \ -42 \ 5 \ -20 \ 56 \ -24 \ 23 \ 77.$$

For those who do not know the encoding matrix A, decoding the cryptogram found in Example 3 is difficult. But for an authorized receiver who knows the encoding matrix A, decoding is relatively simple. The receiver just needs to multiply the coded row matrices by A^{-1} to retrieve the uncoded row matrices [7].

VIII. CONCLUSION

Linear Algebra is the most important part of the mathematics, which contain matrices and its multiplication. Matrix multiplication has some beautiful applications. So in this paper we worked on. First we introduced the matrix multiplication and their definition with some properties. In the second part of the research work, we have tried to introduce population change and population movement by matrix multiplication with examples. Then we described the analysis of traffic flow, which is very important in humane life. Another fantastic application of matrix multiplication is production cost and diplomatic relations among countries, these parts also clarified with some applied examples. As we know the coding theory is one of the important and useful part of the life, hence in this paper also we discussed in the form of matrices, that is encoding and decoding messages.

REFERENCES

[1] Alex Goldvard and Lavi Karp; Applications of Matrices Multiplication to Determinant and Rotations Formulas in R^n. ORT Braude College, Applied Mathematics, Faculty Member, 2010, p.1-2.
[2] David C. Lay; LINEAR ALGEBRA and Its Applications, University of Maryland, Addison-Wesley Publishing Company Ney York, p. 87-89.
[3] David Poole; Linear Algebra a Modern Introduction, 4th Edition. Cengage Learning, USA. 2015, p. 140
[4] Gilbert Strang; Linear Algebra and Its Applications, Fourth Edition. Massachusetts Institute of Technology, USA p. 27-28.
[5] Howard Anton & Charisrorres; Elementary Linear Algebra. Professor Emeritus, Drexel University & University of Pennsylvania. 11th Edition, 2014, p. 30.
[6] L. Spence A. Insel S. Friedberg; Elementary Linear Algebra a Matrix Approach. USA, Second Edition, 2014, p. 6-115.
[7] Ron Larson; Elementary Linear Algebra, Seventh Edition. Brooks/Cole The Pennsylvania State University, The Behrend College USA, 2013, p. 87-88.
[8] Stephen Andrilli and David Hecker; Elementary Linear Algebra, fifth edition, Department of Mathematics and Computer Science La Salle University Philadelphia, PA. Department of Mathematics Saint Joseph’s University Philadelphia, PA. Elsevier, USA, 2016, p. 66-67.
[9] Steven J. Leon; Linear Algebra with Applications, Eighth Edition. University of Massachusetts, Dartmouth, USA. 2010, p. 36-38.
[10] Titu Andreescu; Essential Linear Algebra with Applications, Natural Sciences and Mathematics University of Texas at Dallas, Richardson, TX, USA, Springer Science Business Media New York 2014, p. 16.