Weaknesses and Strengths of the Winkler Method in Designing Foundations

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ABSTRACT

The reaction between the foundation and the soil below it is one of the important issues in civil engineering, and this factor has a significant effect on the behavior of the structure. In this article, first, the basic principles of this method and the simplifications introduced in it are introduced, and then the concept of the most important input parameter of this model, the substrate reaction coefficient, determination relationships, and factors affecting it are discussed. The main goal of this article is to understand the concept of the Winkler method and the bed reaction coefficient so that engineers can choose this model with full knowledge of its weaknesses and strengths and the theories used in it. Also, by studying the factors affecting the bed reaction coefficient, it is possible to estimate More realistic values should be provided so that the results of this method are more correct and accurate.

Keywords- Winkler's method, reaction coefficient, foundation width, environment, parameter.

I. INTRODUCTION

The soil environment has complex behavior due to its nonlinear, stress-dependent, non-isotropic, and non-homogeneous nature. In addition, the behavior of the soil changes according to the layering and characteristics of the load-carrying system, and this problem leads to complexity. The mechanical behavior of the soil and the contact stress distribution are multiplied. Therefore, in order to interpret and simplify soil behavior in soil-foundation interaction problems, models that physically display the characteristics and mechanical properties of the soil environment and are computationally simple are needed, which are called substrate models [1].

These models have appeared since the middle of the 19th century, and in fact, in foundation-soil interaction problems, the relationship between the applied load and the settlements resulting from it is presented in a simpler and more mathematical way by the bed models. One of the most common and simplest of these models is Winkler's method, which is known among most designers [1, 2]. Of course, in addition to that, there are other models that are based on the theories of elasticity and viscoelastic behavior. In this article, due to the wide use and popularity of Winkler's method, attention is paid to this method.

It is hoped that awareness of the concepts, theory, and weak points of this model, as well as paying attention to the factors affecting the reaction coefficient, will be the basis for interpreting the results of computer programs designed based on this method and estimating more accurate values of settlement and pressure. Contact the design engineers to help.

II. WINKLER'S MODEL

Winkler (1867) has considered the soil environment as a system of equally elastic linear springs, independent from both sides, with a distance close to each other but separate from each other, and at each point the relationship between the contact pressure, P, and the settlement, y, the result of which is determined by the reaction coefficient of the bed, K_s , as follows [1]

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$$K_{s} = \frac{P}{Y}$$
(1)

In fact, in this model, the soil under the foundation is replaced by hypothetical springs, the constant of which is K_s .

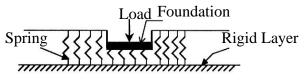


Figure 1: Winkler Foundation model

In terms of structure-soil interaction problems, many researchers, such as Zimmermann (1888), Heteny (1946), Popov (1951), Terzaghi (1932,1955) [3], Vesic (1961) [4], Horvath (1983,1989) [5:6] Daloglu and Vallabhan (2000)] [7] ...

However, the main problem with using this method is determining the numerical value of the bed reaction coefficient or the stiffness of elastic springs.

III. SUBSTITUTE THE REACTION COEFFICIENT AND THE FACTORS AFFECTING IT

This coefficient is the most important parameter of the Winkler method and has the dimension of force per cube of length. Since the physical behavior and mechanical characteristics of the substrate soil are only determined by this, of course, it is necessary to note that there are two types of substrate reaction coefficients.

The coefficient of vertical reaction of the bed and the coefficient of horizontal reaction of the bed [3]. However, in this article, only the vertical reaction coefficient of the bed is discussed, and the meaning of the bed reaction coefficient, K_s , is the vertical reaction coefficient.

Unfortunately, although more than a century ago [3] (1893), Engesser pointed out that the value of K_s in equation 1 decreases with the increase of beam width, B, at the beginning of the 20th century, articles 1 were published that gave unrealistic values for the reaction coefficient. provided a substrate, and the engineers thought that the numerical value of the reaction coefficient depends exclusively on the nature of the substrate, and for an assumed and known substrate, it has a certain and single value [3].

But Terzaghi (1955), in his comprehensive article, investigated the effect of factors affecting K_s separately in flexible horizontal beams and wide foundations, using the works of other researchers, and showed that K_s is nothing but soil characteristics, and in addition, it depends on the nature of the substrate and layering, on the geometrical characteristics of the load-carrying system, and even on the type of incoming load and the distance between the loads, which are

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summarized below the results of his work. In horizontal beams located on the soil surface (striped foundations), one of the factors that have a great effect on the bed reaction coefficient is the width of the foundation, B, and the effect of this factor on the bed reaction coefficient is expressed in two cases:

1. The deformability characteristics of the bed are almost independent of depth changes; in this case, the relation of the bed reaction coefficient changes concerning the footing width is assumed as follows:

$$K_{s} = k_{s1} \frac{B_{1}}{B}$$
(2)

As can be seen from the above relationship, the bed reaction coefficient and footing width have an inverse relationship with each other.

This result can be extended for circular foundations as well.

2. Deformability characteristics are a function of depth, and the modulus of elasticity increases with increasing depth. in this that in relations 2 and 3, K_s and K_{s1} are the reaction coefficients of the bed below the strips of width B and B₁, respectively. Regarding wide foundations, [3]

$$K_{S} = K_{S1} \left(\frac{B + B_{1}}{2B}\right)^{2}$$
 (3)

Westergaard (1926) has determined the reaction between this type of foundation and the soil under it under the effect of concentrated load using the theory of bed reaction and parameter r_0 (hardness radius of slab 2):

$$r_0 = \sqrt[4]{\frac{Eh^3}{12(1-V^2)K_S}}$$
(4)

In this equation, E indicates the coefficient of elasticity of the slab, v indicates the Poisson's ratio of concrete, and h is the thickness of the slab. Ah, the value of v is approximately equal to 0.51 and/or has a unit of length. Under the effect of concentrated load, the majority of the incoming load is transferred to the bed within a radius of 2.5 from the location of the load effect, and outside this area, the settlement of the slab bed is assumed to be insignificant, which is the area of the effect of the concentrated load. It is called, and this range is equivalent to a circular pi with a radius of R.

In this case, the reaction coefficient of the bed is almost equal to the value of the reaction coefficient under the equivalent circular foundation, and in this type of foundation, the reaction coefficient decreases with the increase of the stiffness radius.

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Therefore, in wide foundations under the effect of concentrated load, the reaction coefficient of the bed is independent of the dimensions of the foundation and depends on the range of the effect of concentrated load, R [3]. Of course, Terzaghi has also expanded the results of Westergaard's research for extensive foundations under the effect of concentrated loads, so for more information, refer to reference [3].

The amount of contact pressure under the pi also affects the value of k_s. Because the environment of the elastic soil is not perfect, the resulting stressdeformation diagram is also non-linear, and the k_s value will be different according to different values of stress [8].

The elastic characteristics of the soil (i.e., Es and vs) also affect the value of ks [9, 10]. This effect is more visible in the experimental relationships that are presented to determine ks in the next sections, and it is recommended to influence the effects. Layering and increasing the depth in the numerical value of ks is easier to measure than the changes in the elasticity coefficient with respect to the depth.

The general result of this part is that the bed reaction coefficient is nothing but the characteristics of the soil and depends on external factors other than the nature of the soil, especially the dimensions and geometry of the foundation, which were discussed above, and this is the reason for distinguishing the bed reaction coefficient and the elasticity coefficient. It is soil.

IV. **METHODS OF DETERMINING** THE BED REACTION COEFFICIENT

In general, the methods of determining k_s are: 1: Terzaghi relations [3, 8]2, page loading test [1, 8]3. Experimental relations [4, 9, 10]4, Tahkim test [1, 8] 5, CBR test [1] 6, a triaxial test [1].

In this article, only brief explanations regarding methods 1 and 3 have been provided, and for more information, you can refer to the introduced sources. 4-1: Terzaghi relations

As mentioned in the previous part, Terzaghi has presented some relations according to the effect of the width of the foundation, B, on the reaction coefficient of the substrate (Relations 2 and 3). In these relationships, in order to estimate the value of the bed reaction coefficient, the numerical value of k_{s1} must be known. For this reason, Tarzaghi recommends the use of the bed reaction coefficient under a square plate with a width of k_{s1} (1ft), and by placing B1 = 1ft in relations 2 and 3, the following equations are obtained:

$$k_{s} = k_{s1} (\frac{B+1}{2B})^{2}$$
(5)

Volume-3 Issue-2 || April 2024 || PP. 4-8

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$$K_{S} = K_{S1} \frac{1}{B}$$
(6)

For sandy soils and for clay soils, the suggested values of Terzak for k_{s1} are presented in Table (1) for clay soils and in Table (2) for sandy soils. It should be noted that these relations are written in Fps devices, and when converting the units to SI devices, 1ft must be replaced with 0.3m. Also, these relationships can be used for single piles, strip piles or beams, slabs, and wide piles under the effect of concentrated loads [3].

Of course, it should be noted that relations 6 and 5 are also used in the plate loading test and the value of K_{S1} is determined from the slope of the stress-strain diagram obtained from this test [8].

4-2: Experimental relationships

In the absence of sufficient facilities to conduct the experiment, we can obtain the value of K_S directly by using the empirical relationships provided by researchers for this purpose. One of the first relationships proposed to determine the reaction coefficient was presented by Biot (1937).

$$K_{\rm S} = \frac{0.95E_{\rm S}}{B(1 - V_{\rm S}^{\ 2})} \left(\frac{B^4 E_{\rm S}}{(1 - V_{\rm S}^{\ 2}) {\rm El}}\right)^{0.108} \tag{7}$$

where V_S and E_S are the elastic properties of the soil, B is the footing width, EI is the bending stiffness of the beam, and K_S is the bed reaction coefficient in terms of force per cube of length [9]?

Vesic (1961) has also presented a formula similar to Biot's formula (1937), in which the elastic properties of the substrate (E_s, V_s) and bending stiffness are also used [4].

$$K_{\rm S} = \frac{0.65 E_{\rm S}}{B(1 - V_{\rm S}^2)} \sqrt[12]{\frac{E_{\rm S} B^4}{EI}}$$
(8)

Table 1: Values of ks1 in terms of $\frac{\text{ton}}{\text{ft}^2}$ for square plates with dimensions of 1ft x1ft and strips with a width

of 1 ft located on pre-reinforced clay bed [3]					
Consistency	Rigid	Very	Hard		
of clay	Rigia	Rigid	Thurt		
qu values in terms of ton/ft ³	1-2	2-4	>4		
Limit K _{S1} for square plates	50- 100	100- 200	>200		
Suggested values For square plates	75	150	*300		

* Values greater than $300^{\text{ton}}/\text{ft}^3$ should be estimated by appropriate tests.

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Table 2: ks1 values in terms of $\frac{\text{ton}}{\text{ft}^3}$ for square	
plates with dimensions of 1ft x1ft or	
beams with a width of 1ft located on a sand bed [3].	

beams with a width of fit located on a sand bed [5].					
Relative density of sand	Loose	Medium	Dense		
limit of values k _{s1} For dry and wet sand	20- 60	60-300	300- 1000		
Recommended values for dry and wet sand	40	130	500		
Suggested values for submerged sand	25	80	300		

In addition to these relationships, other empirical relationships have been presented in the articles, which can be referred to in Reference [10] for more information about them. It is also suggested to use the permissible carrying capacity, qa, to estimate the value of K_S , whose relationships are:

$$K_{\rm S} = 40({\rm SF})qa \tag{9}$$

In SI unit and in terms of $\frac{KN}{m^3}$

$$K_{\rm S} = 12(\rm SF)qa \qquad (10)$$

In Fps unit and in terms of $\frac{\text{KN}}{\text{ft}^3}$

where qa is in ksf or kpa. This equation is based on $qa = \frac{q_{ult}}{s_F}$ and the final pressure of the soil in the settlement $\Delta H = 0.0245$ m is obtained. By placing the above values in the formula $K_s = \frac{q_{ult}}{\Delta H}$, the above formulas are obtained. Now, for $\Delta H = 6$, 12, 20mm, the coefficient (40) or (12) can be adjusted as (50) or (16), (83) or (24) and (160) or (84), respectively. The coefficient of 40 is relatively conservative, but smaller hypothetical deformations can also be used [8].

Also, by rewriting the relationship that was presented for determining the settlement of a rectangular foundation on the surface of the elastic half-space with the help of the theory of elasticity by Goodier and Timoshenko (1951), the value of K_S can also be determined.

This equation must be used in the case of flexible foundations located on half space:

$$K_{S} = \frac{E_{S}}{B''(1 - V_{S}^{2})mI_{S}I_{F}}$$
(11)

where E_s and vs are respectively the coefficient of elasticity and Poisson's ratio of the soil, B'is the

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minimum lateral dimension, IF and IS are the effect coefficients that depend on the depth of the foundation, the ratio of the dimensions of the foundation, and V_S , whose values are calculated using the relevant tables and graphs. It can be determined, and m is a coefficient indicator whose value is equal to 1, 2, and 4 for the corner, edge, and center of the foundation, respectively. It is used in context [8].

In the review of technical literature, among the proposed formulas, relations 7, 8, and 11 are more visible [7, 8], and they are introduced as the most common relations for determining the substrate reaction coefficient.

V. WEAKNESSES OF THE WINKLER METHOD

Due to the simplifications introduced in this theory, the behavior shown by the soil is not exactly the same as the real behavior of the soil, and some approximations are introduced in it. These hypotheses create a false impression of linear stress-strain behavior, which is one of the weak points of this theory [3].

Also, according to this theory, the value of the bed reaction coefficient at any point of the surface under the effect of contact pressure has the value of one of the important weaknesses of Winkler's method: the independent assumption of substitute springs in the soil under the foundation. Because by assuming these springs to be independent, we actually assume that the lateral shear stresses do not propagate in the soil, and settlement and deformation occur only within a certain limit due to the incoming loads [1].

Therefore, the important issue in relation to this model is to introduce the behavioral effect of soil cutting into it and to make the springs dependent on each other in order to transfer the lateral strains, in this regard, corrections have been made by the researchers. Another hypothesis that leads to incorrect results from this theory is that, according to this theory, the bending anchor does not expand in the beam or slab located on the Winkler bed under the effect of a uniform load [7].

In addition to the mentioned cases, one of the cases that can cause errors in the reaction theory results is the inaccuracy in determining the numerical value of the substrate reaction coefficient, which can be obtained by using the mentioned methods to determine ks, reliable values. In general, Winkler's method, despite the mentioned weaknesses, has been used by designers for more than a century, has led to acceptable results, especially in the case of flexible beams [2], and is perhaps the only strong point. This method is simple.

VI. CONCLUSION

The bed reaction coefficient is nothing but the characteristics of the soil, and it depends on external factors other than the nature of the soil and especially on

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the dimensions and geometry of the foundation. The only strength of Winkler's method is its computational simplicity, but despite this, it is the most commonly used method among designers and often leads to reasonable and logical results. The most important weakness of Winkler's method is the independent assumption that soil replacement springs under the foundation, and measures should be taken to introduce the behavioral effect of soil shear into it. The important issue in using any behavioral model is to have complete knowledge of its input parameters and accuracy in determination.

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Volume-3 Issue-2 || April 2024 || PP. 4-8

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