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Some Types of Nano Penta Regular Spaces

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ABSTRACT

The goal of our study is to obtain a new space, which we called the Nano Penta regular space, by studying three cases independent of each other in terms of the number of equivalence relationships for the universe set and its subset. Its properties and its relationship to some Nano Penta separation axioms were discussed and the strongly $NpNp$-Regular space and its relationship to the Nano Penta regular space was studied.

***Keywords*-** Nano Penta topological spaces, Nano Penta regular space, Nano Penta separation axioms, Strongly Nano Penta Regular space.

I. INTRODUCTION

In 2013, the notion of Nano-topological space that regarding xx subset of universe GG, which is described as an upper and lower approximation of xx ,this subject introduced by Thivagar M .L. [3].

In 2014, Definition of Nano-Regular space ($N\_\mathfrak{R}N\_\mathfrak{R}$)space is presented by Bhuvaneshwari- M [1] . Nasir [2] in 2020 discussed in his study Nano- Regular space and took the relationship with the separation axioms and also mentioned the definition of Nano subspace.

The Nano Penta topological spaces ($N\mathfrak{F}_{\acute{R}p}$) $N\mathfrak{F}_{\acute{R}p}$) were defined by researchers Rana and El.te. It was presented in 2021 and published in 2023 at the Karbala Conference and defined Np-Homeomorphism, if the mapping are bijective, $NpCont\,NpCont$ and $NpNp$-open[5]. Also she presented definitions of some types of Np-separation axioms through Np-open sets [4].

This paper is devoted to focus on type of $N\mathfrak{F}_{\acute{R}p}$ $N\mathfrak{F}_{\acute{R}p}$ which we named Np-regular and their basic properties are discussed with the suitable theorem and examples and also introduced strongly Np- regular space with some properties. formatted further at JRASB. Define all symbols used in the abstract.

II. PRELIMINARIES

We will provide some definitions necessary for our study.

Definition (2.1) [5] Suppose that ꞠꞠ ≠ φ, where Ꞡ is universe finite set with $\mathfrak{F}_{\acute{R}1}(ɈX)\,\mathfrak{F}_{\acute{R}1}(ɈX)$, $\mathfrak{F}_{\acute{R}2}(ɈX)\,\mathfrak{F}_{\acute{R}2}(ɈX)$, $\mathfrak{F}_{\acute{R}3}(ɈX)\,\mathfrak{F}_{\acute{R}3}(ɈX)$, $\mathfrak{F}_{\acute{R}4}(ɈX)\,\mathfrak{F}_{\acute{R}4}(ɈX)$ and $\mathfrak{F}_{\acute{R}5}(ɈX)\,\mathfrak{F}_{\acute{R}5}(ɈX)$ are five disjoint Nano topologies on ꞠꞠ with regarding ɈXɈX then (Ꞡ, $\mathfrak{F}_{\acute{R}p}(ɈX)$Ꞡ, $\mathfrak{F}_{\acute{R}p}(ɈX)$ is $NpNp$ -topological space ($N\mathfrak{F}_{\acute{R}p}$) $N\mathfrak{F}_{\acute{R}p}$) ,where p=1,2,34,5.

Definition (2.2) [5] A subset ÃÃ of space (Ꞡ, $\mathfrak{F}_{\acute{R}p}$ Ꞡ, $\mathfrak{F}_{\acute{R}p}$ (ɈX)(ɈX)) if Ã = Ĥ ∪ Ǩ ∪ Ḟ ∪ Ṽ ∪ ǰ Ã = Ĥ ∪ Ǩ ∪ Ḟ ∪ Ṽ ∪ ǰ where ĤĤ belong to $\mathfrak{F}_{\acute{R}1}\,\mathfrak{F}_{\acute{R}1}$ (ɈX (ɈX), ǨǨ belong to $\mathfrak{F}_{\acute{R}2}$ (ɈX $\mathfrak{F}_{\acute{R}2}$ (ɈX), Ḟ belong to $\mathfrak{F}_{\acute{R}3}\mathfrak{F}_{\acute{R}3}$ (ɈX)(ɈX)), ṼṼ belong to $\mathfrak{F}_{\acute{R}4}$

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$\mathfrak{F}_{Ŕ4}$ (ℐX) and ǰ belong to $\mathfrak{F}_{Ŕ5}$ (ℐX)$\mathfrak{F}_{Ŕ5}$ (ℐX)) then ÃÃ is called Nano Penta-open set (NpNp_ ÔÔ). Where the union of the five Nano topologies doesn't necessarily have to be on the same topological space so the Nano topological space that fulfils all the intersections and unions of the Np-open sets is called supremum.

Definitions (2.3) [5]

1.If ÃÃ is NpNp-open subset of space (Ǵ, $\mathfrak{F}_{Ŕ_p}$ (ℐX) (Ǵ, $\mathfrak{F}_{Ŕ_p}$ (ℐX)) ,then the complement of ÃÃ is called to Nano Penta-closed set ($NpNp$_ Ĉ) set.

2. The space (Ǵ, $\mathfrak{F}_{Ŕ_p}$ (ℐX))(Ǵ, $\mathfrak{F}_{Ŕ_p}$ (ℐX)) is called $NpNp$-extremely disconnected, if the $NpNp$ -closure of every $NpNp$ _ ÔÔ is $NpNp$ _ ÔÔ set in ǴǴ.

Theorem (2.4)[5] Every Nano open set (N_Ô)N_Ô) set in which of the spaces (Ǵ, $\mathfrak{F}_{Ŕ_i}$ (ℐX)) (Ǵ, $\mathfrak{F}_{Ŕ_i}$ (ℐX)) is $NpNp$_ÔÔ set in (Ǵ, $\mathfrak{F}_{Ŕ_p}$ (ℐX) (Ǵ, $\mathfrak{F}_{Ŕ_p}$ (ℐX)) , where $ii_{=1}$, 2,3,4,5, 2,3,4,5.

Remarks (2.5)

1.The set Ɓ ={Ǵ,∪ $\mathbb{L}_{Ŕp}$(ℐX)Ǵ,∪ $\mathbb{L}_{Ŕp}$(ℐX), BB $_{Ŕ\ p}$ (ℐX)(ℐX)} is the basis for $N\mathfrak{F}_{Ŕ_p}N\mathfrak{F}_{Ŕ_p}$[5].

2.Each relative $N\mathfrak{F}_{Ŕ}$ $N\mathfrak{F}_{Ŕ}$ is relative $N\mathfrak{F}_{Ŕp}$ $N\mathfrak{F}_{Ŕp}$ [5].

3.The $NpNp$Ti-space have topological and hereditary property, where i=0,1,2[4].

Definition (2.6) [5] Suppose that (Ǵ, $\mathfrak{F}_{Ŕ_p}$ (ℐX)) (Ǵ, $\mathfrak{F}_{Ŕ_p}$ (ℐX))and(Ḿ, $\mathfrak{F}_{Ŕp}$ (Y))(Ḿ, $\mathfrak{F}_{Ŕp}$ (Y)) are $N\mathfrak{F}_{Ŕ_p}$ (ℐX)$N\mathfrak{F}_{Ŕ_p}$ (ℐX) with regarding to ℐXℐX and Y. A map Z:(Ǵ, $\mathfrak{F}_{Ŕ_p}$(ℐX))(Ǵ, $\mathfrak{F}_{Ŕ_p}$(ℐX)) → (Ḿ, $\mathfrak{F}_{Ŕ_p}$(Y))Ḿ, $\mathfrak{F}_{Ŕ_p}$(Y))is $NpNp$-continuous ($NpCont$)$NpCont$), if $Z^{-1}Z^{-1}$ (ǰ)of every $NpNp$_ ÔÔ set ǰ in ḾḾ is $NpNp$_ ÔÔ set in ǴǴ.

Definitions (2.7) [4] A space (Ǵ, $\mathfrak{F}_{Ŕ_p}$ (ℐX)) (Ǵ, $\mathfrak{F}_{Ŕ_p}$ (ℐX)) is

1. Nano Penta T_0- space ($NpNp$$T_0$- space) regarding to distinct points â, ƀâ, ƀ ∊ ǴǴ . ∃ $NpNp$ -open set including one of them but not the other.

2. Nano Penta T_1- space ($NpNp$$T_1$-space) regarding to different points â, ƀâ, ƀ belong to ǴǴ, ∃∃ two $NpNp$ _ ÔÔ sets with one of them but not the other.

3. Nano Penta T_2- space($NpNp$$T_2$-space) regarding to different points â, ƀâ, ƀ belong to ǴǴ, ∃∃ two distinct $NpNp$_ ÔÔ sets Ṽ, ǰ ∋ â ∈ Ṽand ƀ ∈ ǰ â ∈ Ṽand ƀ ∈ ǰ.

Definition (2.8) [4]: If (Ǵ, $\mathfrak{F}_{Ŕ_p}$ (ℐX))(Ǵ, $\mathfrak{F}_{Ŕ_p}$ (ℐX)) is $NpNp$-Regular with $NpNp$$T_1$-space then ǴǴ is Np $Np$$T_3$-space.

III. ON NANO PENTA REGULAR SPACES

Now we introduce new definition of Nano Penta regular through the lower, upper approximation and the boundary region of a universe set using an equivalence relation on it.

Definition (3.1) The space (Ǵ, $\mathfrak{F}_{Ŕ_p}$ (ℐX)) Ǵ, $\mathfrak{F}_{Ŕ_p}$ (ℐX)) is said to be Nano Penta Regular space ($Np - \mathfrak{R}$)$Np - \mathfrak{R}$) space if for each Np_Ĉ set Np_Ĉ set Ḟ and a point ũũ not belong to Ḟ ,∃Ḟ ,∃ disjoint two Np_Ô $set$$Np$_Ô sets ĶĶ and ĤĤ in Ǵ Ǵ ∋ũũ belong to Ĥ Ĥ and Ḟ ⊆ ĶḞ ⊆ Ķ.

Example (3. 2) Using five equivalence classes on Ǵ = {1,2,3,4}Ǵ = {1,2,3,4} on an universe set and its with five subsets .let

Ǵ/ Ŕ1 = {{1}, {2,3,4}}Ǵ/ Ŕ1 = {{1}, {2,3,4}} ,
Ǵ/ Ŕ2 = {{3}, {1,2,4}}Ǵ/ Ŕ2 = {{3}, {1,2,4}}
Ǵ/ Ŕ3 = {{2}, {4}, {1,3}}
Ǵ/ Ŕ3 = {{2}, {4}, {1,3}} ,
Ǵ/ Ŕ4 = {{2}, {3}, {1,4}}
Ǵ/ Ŕ4 = {{2}, {3}, {1,4}},
Ǵ/ Ŕ5 = {{4}, {1,2,3}}Ǵ/ Ŕ5 = {{4}, {1,2,3}} ,

we get

$\mathfrak{F}_{Ŕ_p}$ (ℐX) = {Ǵ, φ, {1}, {2,3,4}, {3}, {1,2,4}, {2,4}, {1,3}, {2,3}, {1,4}, {4}, {1,2,3}, {3,4}, {2}, {1,2}, {1,3,4}},so

$(\mathfrak{F}_{Ŕ_p}$ (ℐX)$)^c$ = {Ǵ, φ, {2,3,4}, {1}, {1,2,4}, {3}, {1,3}, [2,4}, {1,4}, {2,3}, {4}{4},{1,2,3}, {2}, {1,2}, {1,3,4}, {3,4}} {1,2,3}, {2}, {1,2}, {1,3,4}, {3,4}}.

Hence (Ǵ, $\mathfrak{F}_{Ŕ_p}$ (ℐX))Ǵ, $\mathfrak{F}_{Ŕ_p}$ (ℐX)) is $Np - \mathfrak{R}$ $Np - \mathfrak{R}$ space.

Theorem (3.3) Each N_$\mathfrak{R}$ N_$\mathfrak{R}$ space is $Np - \mathfrak{R}$ $Np - \mathfrak{R}$ space

Proof. Let (Ǵ, $\mathfrak{F}_{Ŕ_i}$ (ℐX))Ǵ, $\mathfrak{F}_{Ŕ_i}$ (ℐX)) be N_$\mathfrak{R}$ N_$\mathfrak{R}$ space ,where $i = 1,2,3,4,5 i = 1,2,3,4,5$. Suppose that ḞḞ is Nano closed set and a point ũũ not

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belong to $\dot{F}\ in\ Ǵ\ \dot{F}\ in\ Ǵ$. Since $ǴǴ$ be $N\_\mathfrak{R}\ N\_\mathfrak{R}$ space, then ∃ distinct two $N\_\hat{O}\ N\_\hat{O}$ sets $\breve{K}, \hat{H}\ in\ Ǵ$ $\breve{K}, \hat{H}\ in\ Ǵ$ $\ni\ ũ\ \in\ \hat{H}\ \wedge\ \dot{F}\subseteq\breve{K}$ $\ni\ ũ\ \in\ \hat{H}\ \wedge\ \dot{F}\subseteq\breve{K}$ by using theorem (2.4)become $\hat{H},\breve{K}\hat{H},\breve{K}$ are disjoint two $Np_\hat{O}\ set Np_\hat{O}\ set$s ∃∃ ũũ ∈ĤĤ ∧ Ḟ⊆⊆Ǩ. Therefore $(Ǵ, \mathfrak{F}_{Ŕp}(JX))$ $Ǵ, \mathfrak{F}_{Ŕp}(JX))$ is $Np - \mathfrak{R}\ \text{space} Np - \mathfrak{R}\ \text{space}$. $\breve{K}, \hat{H}\ in\ Ǵ\breve{K}, \hat{H}\ in\ Ǵ$ $\ni\ ũ\ \in\ \hat{H}\ \wedge\ \dot{F}\subseteq\breve{K}$ $\ni\ ũ\ \in\ \hat{H}\ \wedge\ \dot{F}\subseteq\breve{K}$ by using theorem (2.4)become $\hat{H},\breve{K}\hat{H},\breve{K}$ are disjoint two $Np_\hat{O}\ set Np_\hat{O}\ set$s ∃∃ ũũ ∈ĤĤ ∧ Ḟ⊆⊆Ǩ. Therefore $(Ǵ, \mathfrak{F}_{Ŕp}(JX))$ $Ǵ, \mathfrak{F}_{Ŕp}(JX))$ is $Np - \mathfrak{R}\ \text{space} Np - \mathfrak{R}\ \text{space}$.

Remark (3.4): The converse of theorem (3.3) is not true. As in the example below.

Example (3.5) $Let\ Ǵ = \{1,2,3,4\}, JX \subseteq Ǵ$ $Let\ Ǵ = \{1,2,3,4\}, JX \subseteq Ǵ$ with five equivalence classes

$Ǵ/Ŕ1 = \{\{1\},\{2,4\},\{3\}\}$
$Ǵ/Ŕ1 = \{\{1\},\{2,4\},\{3\}\}$,
$Ǵ/Ŕ2 = \{\{1\},\{2,3\},\{4\}\}$
$Ǵ/Ŕ2 = \{\{1\},\{2,3\},\{4\}\}$
$Ǵ/Ŕ3 = \{\{1,3,4\},\{2\}\} Ǵ/Ŕ3 = \{\{1,3,4\},\{2\}\}$,
$Ǵ/Ŕ4 = \{\{1,2,3\},\{4\}\}, Ǵ/Ŕ4 = \{\{1,2,3\},\{4\}\}$,
Ǵ/Ŕ5={{1},{4},{2,3}}.Then $\mathfrak{F}_{Ŕp}(JX)\mathfrak{F}_{Ŕp}(JX)$= $\{Ǵ,\varphi,\{1,3\},\{2,4\},\{1\},\{1,2,3\},\{2,3\},\{2\},\{1,3,4\},\{4\},\{1,4\},$ $\{Ǵ,\varphi,\{1,3\},\{2,4\},\{1\},\{1,2,3\},\{2,3\},\{2\},\{1,3,4\},\{4\},\{1,4\},$ $\{3\},\{1,2,4\},\{2,3,4\},\{1,2\}\}$and $(\mathfrak{F}_{Ŕp}(JX))^c = \{Ǵ,\varphi,\{2,4\},\{1,3\},\{2,3,4\},\{4\},\{1,3,4\},$ $\{2\},\{1,4\},\{1,2,3\},\{1,2,4\},\{3\},\{1\},\{3,4\},\{2,3\},\{1,2\}\}$ $\{2\},\{1,4\},\{1,2,3\},\{1,2,4\},\{3\},\{1\},\{3,4\},\{2,3\},\{1,2\}\}$

Hence $(Ǵ, \mathfrak{F}_{Ŕp}(JX))Ǵ, \mathfrak{F}_{Ŕp}(JX))$ is $Np - \mathfrak{R}$ $Np - \mathfrak{R}$ space. But, ∃ at least one of five $\mathfrak{F}_{Ŕ}(JX)$ $\mathfrak{F}_{Ŕ}(JX)$ is not $N\_\mathfrak{R}N\_\mathfrak{R}$ space, it is clear that $(Ǵ, \mathfrak{F}_{Ŕ2}(JX)))(Ǵ, \mathfrak{F}_{Ŕ2}(JX)))$ is not $N\_\mathfrak{R}N\_\mathfrak{R}$ space.

Theorem (3.6): A space $(Ǵ, \mathfrak{F}_{Ŕp}(JX)))$ $(Ǵ, \mathfrak{F}_{Ŕp}(JX)))$ is $Np - \mathfrak{R}Np - \mathfrak{R}$ space if only $\mathfrak{U}_{Ŕp}(JX)\mathfrak{U}_{Ŕp}(JX)$= Ǵ, $\mathbb{L}\mathbb{L}_{Ŕp}(JX)=\varphi$ and $\mathfrak{U}_{Ŕp}(JX) \neq \mathbb{L}_{Ŕp}(JX)\mathfrak{U}_{Ŕp}(JX) \neq \mathbb{L}_{Ŕp}(JX)$.

Proof. If $\mathfrak{U}_{Ŕp}(JX) = Ǵ\mathfrak{U}_{Ŕp}(JX) = Ǵ$ and $\mathbb{L}_{Ŕp}(JX) \neq \varphi\mathbb{L}_{Ŕp}(JX) \neq \varphi$, when $\mathfrak{F}_{Ŕp}(JX) = \{Ǵ,\varphi,\mathbb{L}_{Ŕp}(JX),B_{Ŕp}(JX)\}$, $\mathfrak{F}_{Ŕp}(JX) = \{Ǵ,\varphi,\mathbb{L}_{Ŕp}(JX),B_{Ŕp}(JX)\}$, so $Ǵ,\varphi,\left(\mathbb{L}_{Ŕp}(JX)\right)^c Ǵ,\varphi,\left(\mathbb{L}_{Ŕp}(JX)\right)^c, (B_{Ŕp}(JX))^c$ $(B_{Ŕp}(JX))^c$ are $Np\_\hat{C}\ sets Np\_\hat{C}\ sets$ in $\mathfrak{F}_{Ŕp}(JX)\mathfrak{F}_{Ŕp}(JX)$ which $B_{Ŕp}(JX) = (\mathbb{L}_{Ŕp}(JX))^c B_{Ŕp}(JX) = (\mathbb{L}_{Ŕp}(JX))^c$ and $\mathbb{L}_{Ŕp}(JX) = (B\ Ŕp(JX))^c$ $\mathbb{L}_{Ŕp}(JX) = (B\ Ŕp(JX))^c$ and $ǴǴ$ is extremely disconnected space too , therefore Ǵ is $Np - \mathfrak{R}$ $Np - \mathfrak{R}$ space.

Proposition (3.7) A space $(Ǵ, \mathfrak{F}_{Ŕp}(JX))$ $Ǵ, \mathfrak{F}_{Ŕp}(JX))$ is $N\mathfrak{F}_{Ŕp}\ N\mathfrak{F}_{Ŕp}$ if $\mathfrak{U}_{Ŕp}(JX) \neq Ǵ$ $\mathfrak{U}_{Ŕp}(JX) \neq Ǵ$ and $\mathbb{L}_{Ŕp}(JX) \neq \varphi\mathbb{L}_{Ŕp}(JX) \neq \varphi$ then $(Ǵ, \mathfrak{F}_{Ŕp}(JX))Ǵ, \mathfrak{F}_{Ŕp}(JX))$ is $Np - \mathfrak{R}$ $Np - \mathfrak{R}$ space.

Proof. When $\mathfrak{F}_{Ŕp}(JX)\mathfrak{F}_{Ŕp}(JX)$={Ǵ,φ, $\mathbb{L}_{Ŕp}(JX)$ $\mathbb{L}_{Ŕp}(JX)$, $\mathfrak{U}_{Ŕp}(JX)\mathfrak{U}_{Ŕp}(JX)$, $B_{Ŕp}(JX)$ $B_{Ŕp}(JX)$}, if $\mathfrak{U}_{Ŕp}(JX) \neq Ǵ\mathfrak{U}_{Ŕp}(JX) \neq Ǵ$ and $\mathbb{L}_{Ŕp}(JX)\mathbb{L}_{Ŕp}(JX)$ ≠φ, then $Ǵ,\varphi,(\mathbb{L}_{Ŕp}(JX))^c,(\mathfrak{U}_{Ŕp}(JX))^c$ $Ǵ,\varphi,(\mathbb{L}_{Ŕp}(JX))^c,(\mathfrak{U}_{Ŕp}(JX))^c$ and $(B_{Ŕp}(JX))^c(B_{Ŕp}(JX))^c$ are $Np\_\hat{C}\ set$ $Np_\hat{C}\ set$s in $ǴǴ$. So all these $Np_\hat{C}\ set Np_\hat{C}\ set$s contained only in Ǵ. If taking any point ũũ out of any $Np\_\hat{C}\ set Np\_\hat{C}\ set$. Then only $Np\_\hat{O}\ set$ $Np\_\hat{O}\ set$ which contain the $Np\_\hat{C}\ set Np\_\hat{C}\ set$ is $ǴǴ$ and $ũ \in Ǵũ \in Ǵ$, hence $ǴǴ$ is $Np - \mathfrak{R}$ $Np - \mathfrak{R}$ space.

Example (3.8) Using five equivalence classes on $Ǵ = \{\mathfrak{a},\ell,\mathscr{k},\mathcal{S}\}Ǵ = \{\mathfrak{a},\ell,\mathscr{k},\mathcal{S}\}$ on an universe set and its with five subsets. Let

$Ǵ/Ŕ1 = \{\{\mathfrak{a}\},\{\ell,\mathcal{S}\},\{\mathscr{k}\}\}$
$Ǵ/Ŕ1 = \{\{\mathfrak{a}\},\{\ell,\mathcal{S}\},\{\mathscr{k}\}\}$,
$Ǵ/Ŕ2 = \{\{\mathfrak{a}\},\{\ell,\mathscr{k}\},\{\mathcal{S}\}\}$
$Ǵ/Ŕ2 = \{\{\mathfrak{a}\},\{\ell,\mathscr{k}\},\{\mathcal{S}\}\}$
$Ǵ/Ŕ3 = \{\{\mathfrak{a},\mathscr{k},\mathcal{S}\},\{\ell\}\}$
$Ǵ/Ŕ3 = \{\{\mathfrak{a},\mathscr{k},\mathcal{S}\},\{\ell\}\}$,
$Ǵ/Ŕ4 = \{\{\mathfrak{a},\ell,\mathscr{k}\},\{\mathcal{S}\}\}$
$Ǵ/Ŕ4 = \{\{\mathfrak{a},\ell,\mathscr{k}\},\{\mathcal{S}\}\}$
Ǵ/Ŕ5={{ $\mathfrak{a}\ \mathfrak{a}$ },{ $\mathcal{S}\ \mathcal{S}$ },{ $\ell\ \ell$, $\mathscr{k}\ \mathscr{k}$ }}, we get

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$\mathfrak{F}_{\acute{R}_p}(\mathcal{JX}) = \{ \acute{G}, \varphi, \{a,\ell\}, \{k\}, \{S\}, \{a,\ell,k\}, \{k,S\}, \{a,\ell,S\}\}, \{k,S\}, \{a,\ell,S\}\}$,
$\mathfrak{F}^c{}_{\acute{R}_p}(\mathcal{JX})) = \{\acute{G}, \varphi, \{k,S\}, \{a,\ell,k\}, \{S\}, \{a,\ell\}, \{k\}, \{a,\ell,S\}\}$. Hence($\acute{G}, \mathfrak{F}_{\acute{R}_p}(\mathcal{JX}))$ $\acute{G}, \mathfrak{F}_{\acute{R}_p}(\mathcal{JX}))$ is $Np-\mathfrak{R}Np-\mathfrak{R}$ space. where $\mathfrak{U}_{\acute{R}_p}(\mathcal{JX}) \neq \acute{G}\mathfrak{U}_{\acute{R}_p}(\mathcal{JX}) \neq \acute{G}$, $\mathbb{L}_{\acute{R}_p}(\mathcal{JX}) \neq \varphi$ $\mathbb{L}_{\acute{R}_p}(\mathcal{JX}) \neq \varphi$ and $\mathfrak{U}_{\acute{R}_p}(\mathcal{JX}) = \mathbb{L}_{\acute{R}_p}(\mathcal{JX})$ $\mathfrak{U}_{\acute{R}_p}(\mathcal{JX}) = \mathbb{L}_{\acute{R}_p}(\mathcal{JX})$.

Remark (3.9) If $\mathbb{L}_{\acute{R}_p}(\mathcal{JX}) \neq \varphi\mathbb{L}_{\acute{R}_p}(\mathcal{JX}) \neq \varphi$, $\mathfrak{U}_{\acute{R}_p}(\mathcal{JX}) \neq \acute{G}\mathfrak{U}_{\acute{R}_p}(\mathcal{JX}) \neq \acute{G}$ and $\mathfrak{U}_{\acute{R}_p}(\mathcal{JX}) = \mathbb{L}_{\acute{R}_p}(\mathcal{JX}) = \mathcal{JX}$ $\mathfrak{U}_{\acute{R}_p}(\mathcal{JX}) = \mathbb{L}_{\acute{R}_p}(\mathcal{JX}) = \mathcal{JX}$,then the ($\acute{G}, \mathfrak{F}_{\acute{R}_p}(\mathcal{JX}))\acute{G}, \mathfrak{F}_{\acute{R}_p}(\mathcal{JX}))$ can be $Np-\mathfrak{R}$ $Np-\mathfrak{R}$ space . As given in the following examples.

Examples (3.10)

1.Let $\acute{G}/\acute{R} = \{\{1,2\},\{3\},\{4\}\}$ $\acute{G}/\acute{R} = \{\{1,2\},\{3\},\{4\}\}$ be equivalence class defined on $\acute{G} = \{1,2,3,4\}\acute{G} = \{1,2,3,4\}$ universe sets and five subsets of $\mathcal{JX}\mathcal{JX}$.then $\mathfrak{F}_{\acute{R}_p}(\mathcal{JX})\mathfrak{F}_{\acute{R}_p}(\mathcal{JX})$={$\acute{G}\acute{G}$, $\varphi\varphi$, {1,2},{4},{1,2,3}, {3,4}, {3},{1,2,4}},
$\left(\mathfrak{F}_{\acute{R}_p}(\mathcal{JX})\right)^c = \{\acute{G}, \varphi, \{3,4\}, \{1,2,3\}, \{4\}, \{1,2\}, \{1,2,4\}, \{3\}\}\{1,2\}, \{1,2,4\}, \{3\}\}$.
$\left(\mathfrak{F}_{\acute{R}_p}(\mathcal{JX})\right)^c =$
$\{\acute{G}, \varphi, \{3,4\}, \{1,2,3\}, \{4\}, \{1,2\}, \{1,2,4\}, \{3\}\}$
Hence ($\acute{G}, \mathfrak{F}_{\acute{R}_p}(\mathcal{JX}))\acute{G}, \mathfrak{F}_{\acute{R}_p}(\mathcal{JX}))$ is $Np\_\mathfrak{R}Np\_\mathfrak{R}$ space. When no one of the five $\mathfrak{F}_{\acute{R}}(\mathcal{JX})\mathfrak{F}_{\acute{R}}(\mathcal{JX})$ have $N\_\mathfrak{R}N\_\mathfrak{R}$ space.

2.Let Ǵ={â,ƀ,c̀,d'}with Ǵ/ $\acute{R}_1\acute{R}_1$ ={{ â, ƀ},{ c̀, d'}}, Ǵ/ $\acute{R}_2\acute{R}_2$={{ â},{ ƀ},{ c̀},{ d'}}, Ǵ/ $\acute{R}_3\acute{R}_3$={{ â},{ ƀ, c̀},{ d'}},Ǵ/ $\acute{R}_4\acute{R}_4$={{ â},{ ƀ ,c̀, d'}},Ǵ/ $\acute{R}_5\acute{R}_5$={{ ƀ, d'},{ â},{ c̀}}, hence $\mathfrak{F}_{\acute{R}_p}(\mathcal{JX}$ $\mathfrak{F}_{\acute{R}_p}(\mathcal{JX}$)={Ǵ, φ,{â, ƀ},{ â, ƀ, c̀},{â, d'},{ ƀ, c̀, d'},{â, c̀},{â, ƀ, d'},{â},{ ƀ} {ƀ, c̀},{d'}, {â, c̀, d'},{c̀}}. We get ($\acute{G}, \mathfrak{F}_{\acute{R}_p}(\mathcal{JX}))\acute{G}, \mathfrak{F}_{\acute{R}_p}(\mathcal{JX}))$ is not $Np\_\mathfrak{R}Np\_\mathfrak{R}$ space. when no one of five $\mathfrak{F}_{\acute{R}}(\mathcal{JX})\mathfrak{F}_{\acute{R}}(\mathcal{JX})$ have $N\_\mathfrak{R}$ $N\_\mathfrak{R}$ space.

Remarks (3.11): The space($\acute{G}, \mathfrak{F}_{\acute{R}_p}(\mathcal{JX}))$ $\acute{G}, \mathfrak{F}_{\acute{R}_p}(\mathcal{JX}))$ then :

First state: $Np\_\mathfrak{R}\ Np\_\mathfrak{R}$ space only if the five $\mathfrak{F}_{\acute{R}}(\mathcal{JX})\mathfrak{F}_{\acute{R}}(\mathcal{JX})$are $N\_\mathfrak{R}N\_\mathfrak{R}$ space. As is the theorem(3.2)

Second state: $Np\_\mathfrak{R}Np\_\mathfrak{R}$ space only if at least one of five $\mathfrak{F}_{\acute{R}}(\mathcal{JX})\ \mathfrak{F}_{\acute{R}}(\mathcal{JX})$ is $N\_\mathfrak{R}N\_\mathfrak{R}$ space. As is the example (3.5).

Third state: can be $Np\_\mathfrak{R}Np\_\mathfrak{R}$ space, if there's no one of the five $\mathfrak{F}_{\acute{R}}(\mathcal{JX})\ \mathfrak{F}_{\acute{R}}(\mathcal{JX})$ is $N\_\mathfrak{R}N\_\mathfrak{R}$ space. As is the example (3.10).

<u>**Result**</u> : we note that three cases are independent of each other in terms of structural formation of $Np\_\mathfrak{R}Np\_\mathfrak{R}$ space.

Proposition (3.12) The following conditions in space ($\acute{G}, \mathfrak{F}_{\acute{R}_p}(\mathcal{JX}))\acute{G}, \mathfrak{F}_{\acute{R}_p}(\mathcal{JX}))$ are equivalent:

1. Ǵ is $NpNp$-Regular.
2. For each point ũ belong to Ǵ and each $NpNp$-Ô set $\hat{H}_1\hat{H}_1$ including ũ, ∃ $\hat{H}_2\ \hat{H}_2$ is $NpNp$-Ô set ∋ ũ belong to $\hat{H}_2\hat{H}_2 \subseteq NpClNpCl(\hat{H}_2\hat{H}_2) \subseteq \hat{H}_1\hat{H}_1$.
3. For each $NpNp$-Ĉ set Ḟ the intersection of all the Np Np-neighborhoods of Ḟ is Ḟ.
4. For each set Ḟ and Ǩ is $NpNp$- Ô set ∋ ḞḞ∩ǨǨ≠φ , ∃ a $NpNp$- Ô set Ṽ such that Ḟ∩Ṽ ≠φ and $NpCl($ $NpCl($Ṽ) ⊆Ǩ .
5. for each Ḟ≠φ and $NpNp$- Ĉ set Ǩ ∋ Ḟ∩Ǩ=φ, ∃ distinct $NpNp$- Ô sets $\hat{H}\hat{H}$ and ĵ ∋ ḞḞ∩$\hat{H}\hat{H}$≠φ and ǨǨ⊆ĵĵ .

Proof .

From 1 to 2: Suppose that $\hat{H}_1\hat{H}_1$ is $NpNp$- Ô set including ũ. Then Ǵ-$\hat{H}_1\hat{H}_1$ is $NpNp$- Ĉ set and ũ ∉Ǵ-$\hat{H}_1\hat{H}_1$. Since Ǵ is $NpNp$-Regular space, ∃ Ã and $\hat{H}_2$ $\hat{H}_2$ are $NpNp$- Ô sets such that Ǵ-$\hat{H}_1\hat{H}_1$⊆Ã, ũ∈$\hat{H}_2\hat{H}_2$ and Ã∩$\hat{H}_2\hat{H}_2$=φ , so $NpClNpCl$(Ǵ-Ã) =Ǵ-Ã for $\hat{H}_2$ $\hat{H}_2$⊆Ǵ-Ã and Ǵ-Ã is $NpNp$- Ĉ set. Therefore $NpCl$ $NpCl(\hat{H}_2\hat{H}_2) \subseteq \hat{H}_1\hat{H}_1$.

From 2 to 3 : Suppose that Ḟ is a $NpNp$- Ĉ set and ũ ∉Ḟ .Then , Ǵ-Ḟ is $NpNp$- Ô set and contains ũ. By part (2) then $\hat{H}_2\hat{H}_2$ is $NpNp$- Ô set ∋ ũ∈$\hat{H}_2\hat{H}_2 \subseteq NpCl(\hat{H}_2)$ $NpCl(\hat{H}_2)$⊆Ǵ-Ḟ and Ǵ-$\hat{H}_2\hat{H}_2$ ⊇Ǵ- $NpCl(\hat{H}_2)$ $NpCl(\hat{H}_2)$⊇ Ḟ.

Consequently, Ǵ-$\hat{H}_2\hat{H}_2$ is $NpNp$-neighborhood of Ḟ that ũ does not belong .So (3) comes true.

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From 3 to 4 : Assume that Ḟ∩Ǩ ≠φ and Ǩ is $NpNp$- Ô set and ũ ∈ Ḟ∩Ǩ. Since ũ ∉ $NpNp$- Ĉ set Ǵ-Ǩ ,∃ a Np Np-neighborhood of Ǵ-Ǩ, $\hat{H}_1\hat{H}_1$ ∋ ũ ∉ $\hat{H}_1.\hat{H}_1$. Suppose Ǵ- Ǩ ⊆ $\hat{H}_2\hat{H}_2$ ⊆ $\hat{H}_1\hat{H}_1$,where $\hat{H}_2\hat{H}_2$ is $NpNp$- Ô set. Then Ṽ = Ǵ- $\hat{H}_1\hat{H}_1$ is $NpNp$- Ô set which includes ũ and Ḟ∩Ṽ ≠φ. So, Ǵ- $\hat{H}_2\hat{H}_2$ ∈ $NpNp$- Ĉ, $NpClNpCl$ (Ṽ) =$NpClNpCl$ Ǵ - $\hat{H}_1\hat{H}_1$ ⊆ Ǵ- $\hat{H}_2\hat{H}_2$ ⊆ Ǩ .

From 4 to 5 : If Ḟ ∩ Ǩ =φ, when Ḟ ≠φ and Ǩ is $NpNp$- Ĉ set, then Ḟ ∩ (Ǵ- Ǩ) ≠φ and Ǵ- Ǩ is $NpNp$- Ô set. So that by part (5), ∃ a $NpNp$- Ô set $\hat{H}\hat{H}$ ∋ Ḟ ∩ $\hat{H}\hat{H}$ ≠φ , $\hat{H}\hat{H}$ ⊆ $NpCl\ (\hat{H})\ NpCl\ (\hat{H})$ ⊆ Ǵ- Ǩ. put ǰ = Ǵ -Np Np_{Cl} $(\hat{H})(\hat{H})$.Then Ǩ ⊆ ǰ and $\hat{H}\hat{H}$, ǰ are $NpNp$- Ô sets, ∃ ǰ = Ǵ-$NpCl\ \hat{H}NpCl\ \hat{H}$ ⊆ Ǵ- $\hat{H}\hat{H}$.

From 5 to 1 : Obvious.

Proposition (3.13) A space $(Ǵ,\mathfrak{F}_{Ŕp}\ (Ꭓ))$ $(Ǵ,\mathfrak{F}_{Ŕp}\ (Ꭓ))$ is N $\mathfrak{F}_{Ŕp}$ $\mathfrak{F}_{Ŕp}$ if ǴǴ is $Np\_\mathfrak{R}$ $Np\_\mathfrak{R}$ space, then for each ũũ ∈ ǴǴ and $\hat{H}_1$ is $Np\_\hat{O}\ set\ Np\_\hat{O}\ set$ containing ũũ, ∃ $\hat{H}_2\hat{H}_2$ is $Np\_\hat{O}\ set\ Np\_\hat{O}\ set$ containing ũũ ∃ ũ ∈ $\hat{H}_2$ ∃ ũ ∈ $\hat{H}_2$ ⊆⊆ $\overline{\hat{H}}_2\overline{\hat{H}}_2$ ⊆ $\hat{H}$⊆ $\hat{H}$.

Proof . Suppose that ǴǴ is $Np\_\mathfrak{R}Np\_\mathfrak{R}$ space ,so ũũ ∈∈ ǴǴ and $\hat{H}_1\hat{H}_1$ is $Np\_\hat{O}\ set Np\_\hat{O}\ set$ including ũũ → $Ǵ\_\hat{H}_1Ǵ\_\hat{H}_1$ is $Np\_\hat{C}\ setNp\_\hat{C}\ set$ ∋ ũũ ∈∈ $Ǵ\_\hat{H}_1Ǵ\_\hat{H}_1$,then ∃ distinct two $Np\_\hat{O}\ sets$ $Np\_\hat{O}\ sets$ $\hat{H}_2\hat{H}_2$, Ǩ ∋ ũǨ ∋ ũ ∈∈ $\hat{H}_2\hat{H}_2$ ∧∧ Ǵ Ǵ $\hat{H}_1\hat{H}_1$⊆Ǩ then Ǵ_ǨǴ_Ǩ ⊆$\hat{H}_1\hat{H}_1$, when $\hat{H}_2\hat{H}_2$ ∩ Ǩ ∩ Ǩ = $\varphi \rightarrow NpCl(\hat{H}_2)$ ∩ Ǩ$\varphi \rightarrow NpCl(\hat{H}_2)$ ∩ Ǩ = $\varphi \rightarrow NpCl\varphi \rightarrow NpCl$($\hat{H}\hat{H}_2$).Hence ũ ∈ $\hat{H}_2$ ⊆ $\overline{\hat{H}_2}$ ⊆ $\hat{H}_1$.ũ ∈ $\hat{H}_2$ ⊆ $\overline{\hat{H}_2}$ ⊆ $\hat{H}_1$.

Theorem (3.14) A space ($(Ǵ,\mathfrak{F}_{Ŕp}\ (Ꭓ))(Ǵ,\mathfrak{F}_{Ŕp}\ (Ꭓ))$ is $N\mathfrak{F}_{Ŕp}$ $N\mathfrak{F}_{Ŕp}$ in the following conditions are equivalent.

1. ǴǴ is $Np - RNp - R$ space.
2. Every $Np\_\hat{O}\ set Np\_\hat{O}\ set$ $\hat{H}\hat{H}$ is a Union of Npr Npr sets .
3. Every $Np\_\hat{C}\ setNp\_\hat{C}\ set$ BB is a intersection of $NprNpr$ sets.

Proof. From (1) to (2) : Let $\hat{H}\hat{H}$ be a $Np\_\hat{O}\ set$ $Np\_\hat{O}\ set$, a point ũ ∈ $\hat{H}$,$Then\ \tilde{A} = Ǵ\backslash\ \hat{H}$ ũ ∈ $\hat{H}$,$Then\ \tilde{A} = Ǵ\backslash\ \hat{H}$ is $Np\_\hat{C}\ setNp\_\hat{C}\ set$ by hypothesis ,∃ two distinct $Np\_\hat{O}\ set\ W_1\ and\ W_2\ of\ Ǵ$ $Np\_\hat{O}\ set\ W_1\ and\ W_2\ of\ Ǵ$,such that ũ ⊆ W_2 ∧ Ã ⊆ W_1ũ ⊆ W_2 ∧ Ã ⊆ W_1, if $\hat{H} = NpCl(W_2)\ then$ Ḟ is $\hat{H} = NpCl(W_2)\ then$ Ḟ $is\ Np\_\hat{C}\ set\ Np\_\hat{C}\ set$ ∧ Ḟ∩ Ã⊆ Ḟ ∩ W_1Ḟ∩ Ã⊆ Ḟ ∩ W_1. So, ũ ∈Ḟ ⊆ $\hat{H}$ũ ∈Ḟ ⊆ $\hat{H}$,thus $\hat{H}\hat{H}$ is union of Npr Npr sets.

From (2) to (3):obvious

From (3) to (1): Let BB be $Np\_\hat{C}\ set\ Np\_\hat{C}\ set$ ∧ ũ∉ BB, ∃ ḞḞ is $NprNpr$ set ∃ Ã ⊆ Ḟ∋ Ã ⊆ Ḟ ∧ ũ ∉ Ḟ ,if $\hat{H}$ = Ǵ\Ḟ ũ ∉ Ḟ ,if $\hat{H}$ = Ǵ\Ḟ , then $\hat{H}\hat{H}$ is $Np\_\hat{O}\ set\ Np\_\hat{O}\ set$ containing ũ ũ ∧ $\hat{H}\hat{H}$ ∩Ã=φ, then ($(Ǵ,\mathfrak{F}_{Ŕp}\ (Ꭓ))(Ǵ,\mathfrak{F}_{Ŕp}\ (Ꭓ))$ is $Np\_\mathfrak{R}$ $Np\_\mathfrak{R}$ space.

By adding some conditions to the function we get the $Np\_\mathfrak{R}Np\_\mathfrak{R}$ space that can be moved and raised.

Theorem (3.15) Let (Ǵ, $\mathfrak{F}\mathfrak{F}$ $_{Ŕ p}$(Ꭓ)) be $Np\_\mathfrak{R}Np\_\mathfrak{R}$ space, a surjective $\Upsilon: (Ǵ,\mathfrak{F}_{Ŕp}\ (Ꭓ)) \rightarrow (Ḿ,\mathfrak{F}_{Ŕp}\ (Ɏ$ $\Upsilon: (Ǵ,\mathfrak{F}_{Ŕp}\ (Ꭓ)) \rightarrow (Ḿ,\mathfrak{F}_{Ŕp}\ (Ɏ)$ is $Np\ Cont$ $Np\ Cont$ and $NpNp$- open function, then $(Ḿ,\mathfrak{F}_{Ŕp}\ (Ɏ))\ (Ḿ,\mathfrak{F}_{Ŕp}\ (Ɏ))$ is $Np\_\mathfrak{R}Np\_\mathfrak{R}$ space.

Proof. Let ḞḞ be any $Np\_\hat{C}\ Np\_\hat{C}$ subset of Ḿ and a point ũ ∈ Ḿ with ũ ∉ Ḟ , ũ ∉ Ḟ , then ϒϒ (ũ) =ṽ ṽ when ṽ ∈ Ǵ ṽ ∈ Ǵ, ṽṽ = ϒϒ $^{-1}$ (ũ) (because ϒ ϒ is surjective). By hypothesis, ∃ $\hat{H}$,Ǩ $\hat{H}$,Ǩ are $Np\_\hat{O}$ $Np\_\hat{O}$ sets ∃ ṽ ∈ $\hat{H}\ and\ \Upsilon^{-1}$(Ḟ) ⊆ Ǩ ∃ ṽ ∈ $\hat{H}\ and\ \Upsilon^{-1}$(Ḟ) ⊆ Ǩ,when ϒ ϒare$Np -$ $Np -$open surjective. We get ũ ∈ $\Upsilon(\hat{H})$ũ ∈ $\Upsilon(\hat{H})$ and Ḟ ⊆ ϒ (Ǩ)Ḟ ⊆ ϒ (Ǩ) so, $\Upsilon(\hat{H}) \cap \Upsilon$(Ǩ) = $\Upsilon(\hat{H}$ ∩ Ǩ) = $\Upsilon(\varphi)$ $\Upsilon(\hat{H}) \cap \Upsilon$(Ǩ) = $\Upsilon(\hat{H}$ ∩ Ǩ) = $\Upsilon(\varphi)$=$\varphi\varphi$.Hence $(Ḿ,\mathfrak{F}_{Ŕp}\ (Ɏ))\ (Ḿ,\mathfrak{F}_{Ŕp}\ (Ɏ))$ is $Np\_\mathfrak{R}\ Np\_\mathfrak{R}$ space.

Theorem(3.16) Let$(Ḿ,\mathfrak{F}_{Ŕp}\ (Ɏ))\ (Ḿ,\mathfrak{F}_{Ŕp}\ (Ɏ))$ be $Np\_\mathfrak{R}Np\_\mathfrak{R}$ space, if injective $\Upsilon: (Ǵ,\mathfrak{F}_{Ŕp}\ (Ꭓ)) \rightarrow (Ḿ,\mathfrak{F}_{Ŕp}\ (Ɏ))$

Ɏ: (Ǵ, $\mathfrak{F}_{\acute{R}p}$ (Ꭓ)) → (Ḿ, $\mathfrak{F}_{\acute{R}p}$ (Ɏ))
Ɏ: (Ǵ, $\mathfrak{F}_{\acute{R}p}$ (Ꭓ)) → (Ḿ, $\mathfrak{F}_{\acute{R}p}$ (Ɏ)) is *Np Cont Np Cont*, *NpNp*- closed mapping then (Ǵ, $\mathfrak{F}_{\acute{R}p}$ (Ꭓ)) (Ǵ, $\mathfrak{F}_{\acute{R}p}$ (Ꭓ)) is *Np_ℜNp_ℜ* space.

Proof. A point ũũ is not belong ḞḞ is *Np_ĈNp_Ĉ* subset of ǴǴ,since ɎɎ is injective. Then ɎɎ (Ḟ) is *Np Np*- Ĉ set in ḾḾ ∋ ɎɎ (ũ)∉ Ɏ Ɏ (Ḟ) so ,∃ two distinct *Np_Ô sets* Ĥ ,Ǩ *in* Ḿ *Np_Ô sets* Ĥ ,Ǩ *in* Ḿ ∋
Ɏ(ũ) ∈ Ĥ *and* Ɏ(Ḟ) ⊆ Ǩ
Ɏ(ũ) ∈ Ĥ *and* Ɏ(Ḟ) ⊆ Ǩ (because ḾḾ is *Np_ℜ Np_ℜ* space). Therefore, ɎɎ $^{-1}$ (ĤĤ)∧ ɎɎ $^{-1}$ (Ǩ) are *Np_Ô set Np_Ô set* in ǴǴ,
ũ ∈ $Ɏ^{-1}$(Ĥ) ∧ Ḟ ⊆ $Ɏ^{-1}$(Ǩ)
ũ ∈ $Ɏ^{-1}$(Ĥ) ∧ Ḟ ⊆ $Ɏ^{-1}$(Ǩ)∧
$Ɏ^{-1}$ (Ĥ) ∩ $Ɏ^{-1}$ (Ǩ) = φ
$Ɏ^{-1}$ (Ĥ) ∩ $Ɏ^{-1}$ (Ǩ) = φ.Hence Ǵ is *Np_ℜ Np_ℜ* space.

Proposition (3.17) A space (Ǵ, $\mathfrak{F}_{\acute{R}i}$ (Ꭓ)) (Ǵ, $\mathfrak{F}_{\acute{R}i}$ (Ꭓ)) is *N_ℜN_ℜ* space, if a surjective Ɏ:Ɏ: (Ǵ, $\mathfrak{F}_{\acute{R}i}$ (Ꭓ)) → (Ḿ, $\mathfrak{F}_{\acute{R}i}$ (Ɏ))
(Ǵ, $\mathfrak{F}_{\acute{R}i}$ (Ꭓ)) → (Ḿ, $\mathfrak{F}_{\acute{R}i}$ (Ɏ))is *NCont NCont* and *NN*-open mapping, then ḾḾ is *Np_ℜ Np_ℜ* space ,where i=1,2,3,4,5 .

Proof. Suppose that Ḟ be a *N_ Ĉ setN_ Ĉ set* in ǴǴ also ῢ ∉ Ḟῢ ∉ Ḟ ∧ ῢ = Ɏ(ũɎ(ũ) for some ũ∈ Ǵ∈ Ǵ ,when Ɏ Ɏ is *NCont NCont* we get $Ɏ^{-1}Ɏ^{-1}$ (Ḟ) is *N_ Ĉ set N_ Ĉ set* in Ǵ ∃ ũ ∉ $Ɏ^{-1}$ (Ḟ ∃ ũ ∉ $Ɏ^{-1}$ (Ḟ). Since Ǵ is *N_ℜ N_ℜ* space, then ∃ ∃ Ĥ *and* ǨĤ *and* Ǩ are disjoint two *N_Ô N_Ô* sets ∋ ũ ∈ Ĥ ∋ ũ ∈ Ĥ ∧ $Ɏ^{-1}Ɏ^{-1}$ (Ḟ) ⊆Ǩ. That is ῢ ∈ Ɏ (Ĥ)ῢ ∈ Ɏ (Ĥ)∧ Ḟ ∈ Ɏ (Ǩ)Ḟ ∈ Ɏ (Ǩ) ,where Ɏ Ɏ is *NN*-open function in Ḿ Ḿ, ɎɎ (ĤĤ) ∩∩ ɎɎ (Ǩ) = ɎɎ (Ĥ ∩Ĥ ∩Ǩ)=φ. By using theorem (2.4) So,ḾḾ is *Np_ℜNp_ℜ*space.

Proposition (3.18): A space (Ḿ, $\mathfrak{F}_{\acute{R}i}$ (Ɏ)) (Ḿ, $\mathfrak{F}_{\acute{R}i}$ (Ɏ)) is *N_ℜN_ℜ* space, if injective Ɏ: (Ǵ, $\mathfrak{F}_{\acute{R}i}$ (Ꭓ)) → (Ḿ, $\mathfrak{F}_{\acute{R}i}$ (Ɏ))
Ɏ: (Ǵ, $\mathfrak{F}_{\acute{R}i}$ (Ꭓ)) → (Ḿ, $\mathfrak{F}_{\acute{R}i}$ (Ɏ))is *NCont NCont* and *NN*- closed function, then ǴǴ is *Np_ℜ Np_ℜ* space, where i=1,2,3,4,5.

Proof. Suppose that ḞḞ is *N_ Ĉ setN_ Ĉ set* in ǴǴ and ũũ not belong to ḞḞ, where ɎɎ is *NN*- closed function , ɎɎ (Ḟ) is *N_ Ĉ setN_ Ĉ set* in ḾḾ∋ ɎɎ (ũ) not belong to ɎɎ (Ḟ). Since Ḿ Ḿ is *N_ℜ N_ℜ* space, then ∃ two distinct *N_Ô setsN_Ô sets* Ĥ *and* Ǩ Ĥ *and* Ǩ ∋ Ɏ(ũ)Ɏ(ũ) ⊆ ĤĤ ∧ ɎɎ(Ḟ)⊆ Ǩ Ǩ,also ũ ∈ ($Ɏ^{-1}$ (Ĥ)) ũ ∈ ($Ɏ^{-1}$ (Ĥ))∧
Ḟ ⊆ $Ɏ^{-1}$(Ǩ), Ḟ ⊆ $Ɏ^{-1}$(Ǩ), where Ɏ Ɏ is *NCont , NCont ,* by using theorem (2.4) thus
$Ɏ^{-1}$ (Ĥ) ∩ $Ɏ^{-1}$ (Ǩ) = φ
$Ɏ^{-1}$ (Ĥ) ∩ $Ɏ^{-1}$ (Ǩ) = φ. Then ǴǴ is *Np_ℜ Np_ℜ* space.

Proposition (3.20) Every *NpNp*$_{T_0}$-space and *Np_ℜ Np_ℜ* space is *NpNp*$_{T_1}$- space.

Proof. Suppose that (Ǵ, $\mathfrak{F}_{\acute{R}p}$ (Ꭓ))(Ǵ, $\mathfrak{F}_{\acute{R}p}$ (Ꭓ)) are *Np_ℜ Np_ℜ* space and *NpNp*$_{T_0}$-space , then ∀ {ũ} {ũ} is *Np_Ĉ Np_Ĉ* subset of Ǵ Ǵ, for all ũ ∈ũ ∈ ǴǴ. So ῢ be any point of ǴǴ \{ũ\{ũ}, then ũ ≠ ῢ ũ ≠ ῢ , ∃ Ĥ, Ǩ∃ Ĥ, Ǩ are disjoint two *Np_Ô sets* ∋ {ũ} ⊆ Ǩ*Np_Ô sets* ∋ {ũ} ⊆ Ǩ ∧ ῢ ∈ Ĥῢ ∈ Ĥ implies that ῢ ∈ Ĥ ∧ ũ ∈ Ǩ ῢ ∈ Ĥ ∧ ũ ∈ Ǩ. Then (Ǵ, $\mathfrak{F}_{\acute{R}p}$ (Ꭓ))(Ǵ, $\mathfrak{F}_{\acute{R}p}$ (Ꭓ)) is *NpNp*$_{T_1}$-space.

Proposition (3.21): Every *Np_ℜNp_ℜ* space and *Np Np*$_{T_1}$-space is *NpNp*$_{T_2}$-space.

Proof. A space (Ǵ, $\mathfrak{F}_{\acute{R}p}$ (Ꭓ))(Ǵ, $\mathfrak{F}_{\acute{R}p}$ (Ꭓ)) is a *Np_ℜNp_ℜ*, *NpNp*$_{T_1}$-space. To prove (Ǵ, $\mathfrak{F}_{\acute{R}p}$ (Ꭓ))(Ǵ, $\mathfrak{F}_{\acute{R}p}$ (Ꭓ)) is a *NpNp*$_{T_2}$-space. Let ũ, ῢũ, ῢ∈ Ǵ Ǵ ∃∃ ũ ≠ ῢ ũ ≠ ῢ ∧ ǴǴ is *Np Np*$_{T_1}$-space, so{ũũ},{ῢ} are *NpNp*- Ĉ in ǴǴ, then Ǵ Ǵ \{ ũ}\{ ũ}, Ǵ Ǵ \{ῢ\{ῢ } are *Np_Ô sets Np_Ô sets* . Moreover ῢ ∈ Ǵ Ǵ \ {ũ}\ {ũ},for ũũ≠ ῢ. Since ῢ ∈ Ǵ Ǵ \{ũ}, Ǵ\{ ũ}\{ũ}, Ǵ\{ ũ} is *Np_Ô setsNp_Ô sets* in ǴǴ ,then ∃ ǴǴ \{ũ}\{ũ} is *Np* − neighborhood Ĥ *Np* − neighborhood Ĥ of ῢ∋ ῢ∈ $\overline{ĤĤ}$⊆ Ǵ Ǵ \{ũ

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\{ũ}.Thus UU = Ĥ⁰, ṼĤ⁰, Ṽ = ǴǴ −− $\overline{Ĥ}\overline{Ĥ}$ (because Ǵ Ǵ is $Np\_\mathfrak{R}Np\_\mathfrak{R}$). It's clear U , ṼU , Ṽ are $Np\_\hat{O}\ set\ Np\_\hat{O}\ set$ in ǴǴ. U ∩ ṼU ∩ Ṽ = Ĥ⁰ ∩ (Ǵ − Ĥ⁰ ∩ (Ǵ − $\overline{Ĥ}\overline{Ĥ}$) = ĤĤ₀∩ (G − Ĥ⁰ ∩ (G − Ĥ⁰) ∩ $\overline{Ĥ}\overline{Ĥ}$ = $\varphi\varphi$. So, U = Ĥ⁰, U = Ĥ⁰ , ĤĤ is $NpNp$ - neighborhoods of v̈ → v̈ ∈ Uv̈ ∈ U , Ṽ = Ǵ − Ǵ − $\overline{Ĥ}\overline{Ĥ}$, $\overline{Ĥ}\overline{Ĥ}$ ⊆ Ǵ − Ǵ − { ũ } → {ũ} ⊆ Ǵ − $\overline{Ĥ}$ = Ṽ {ũ} ⊆ Ǵ − $\overline{Ĥ}$ = Ṽ → {ũ} ⊆ Ṽ → ũ ∈ Ṽ. {ũ} ⊆ Ṽ → ũ ∈ Ṽ. We get ũũ, v̈ ∈v̈ ∈ ǴǴ ∋ ũ ≠ v̈∋ ũ ≠ v̈ , ∃∃ $Np\_\hat{O}\ setsNp\_\hat{O}\ sets$ U, ṼU, Ṽ ⊆ ǴǴ ∋∋ ũ ∈ Ṽũ ∈ Ṽ, v̈ v̈ ∈ U∈ U .Thus (Ǵ, $\mathfrak{F}_{\acute{R}p}$ (ℐ)) (Ǵ, $\mathfrak{F}_{\acute{R}p}$ (ℐ)) is $NpTNpT$ ₂ -space.

Theorem (3.22) The property of a space being a $Np\_\mathfrak{R}$ $Np\_\mathfrak{R}$ space is $NpNp$ –topological property.

Proof. Let ff: (Ǵ, $\mathfrak{F}_{\acute{R}p}$ (ℐ))(Ǵ, $\mathfrak{F}_{\acute{R}p}$ (ℐ)) → (Ḿ, $\mathfrak{F}_{\acute{R}p}$ (Ɏ)) → (Ḿ, $\mathfrak{F}_{\acute{R}p}$ (Ɏ)) be $NpNp$_ homeomorphism and (Ǵ, $\mathfrak{F}_{\acute{R}p}$ (ℐ))(Ǵ, $\mathfrak{F}_{\acute{R}p}$ (ℐ)) be $Np\_\mathfrak{R}Np\_\mathfrak{R}$ space. To prove (Ḿ, $\mathfrak{F}_{\acute{R}p}$ (Ɏ)) (Ḿ, $\mathfrak{F}_{\acute{R}p}$ (Ɏ)) is $Np\_\mathfrak{R}Np\_\mathfrak{R}$ space. Let v̈ ∈ Ḿ v̈ ∈ Ḿ and Ḟ be $Np\_\hat{C}\ Np\_\hat{C}$ subset of ḾḾ such that v̈ ∉ Ḟv̈ ∉ Ḟ, ff is bijective . Then ∃ ũ ∈ Ǵ ∃ ũ ∈ Ǵ, f(ũ) = v̈f(ũ) = v̈ and ũ = fũ = f₋₁(v̈)(v̈). Since ff and ff₋₁ both are $NpCont$ $NpCont$, we get ff₋₁ (Ḟ) ⊆Ḟ) ⊆ ǴǴ is $Np\_\hat{C}\ set$ $Np\_\hat{C}\ set$, v̈ ∉ Ḟv̈ ∉ Ḟ → ff₋₁ (v̈)(v̈) ∉ f∉ f₋₁ (ḞḞ)→ ũũ ∉∉ ff₋₁(ḞḞ). Now, ũ ∈ Ǵũ ∈ Ǵ, ũ ∉ f ũ ∉ f₋₁ (ḞḞ) and ff₋₁ (Ḟ)(Ḟ) is $Np\_\hat{C}\ set$ $Np\_\hat{C}\ set$ in Ǵ Ǵ. By using definition, ∃ Ǩ, Ĥ Ǩ, Ĥ disjoint two $Np\_\hat{O}\ setsNp\_\hat{O}\ sets$ in ǴǴ Ǩ ∩ Ĥ Ǩ ∩ Ĥ = $\varphi\varphi$. Thus, ũ ∈ Ǩũ ∈ Ǩ, ff₋₁ (ḞḞ) ⊆ Ĥ ⊆ Ĥ → f(ũ) ∈ ff(ũ) ∈ f (ǨǨ) and ff(ff₋₁ (ḞḞ)) ⊆ ff(ĤĤ), f(Ǩ ∩ Ĥ)f(Ǩ ∩ Ĥ) = $f(\varphi) = \varphi$ $f(\varphi) = \varphi$. Furthermore, it, v̈ ∈ f(Ǩ)v̈ ∈ f(Ǩ), Ḟ ⊆ f(Ĥ)Ḟ ⊆ f(Ĥ) f(Ǩ) ∩ f(Ĥf(Ǩ) ∩ f(Ĥ) = $\varphi\varphi$, Then f(Ǩ), f(Ĥ)f(Ǩ), f(Ĥ) are two $Np\_\hat{O}\ setsNp\_\hat{O}\ sets$ in $\mathfrak{F}_{\acute{R}p}\mathfrak{F}_{\acute{R}p}$. Hence (Ḿ, $\mathfrak{F}_{\acute{R}p}$ (Ɏ))(Ḿ, $\mathfrak{F}_{\acute{R}p}$ (Ɏ)) is $Np\_\mathfrak{R}Np\_\mathfrak{R}$ space.

Theorem (3.23) Every subspace of $Np\_\mathfrak{R}Np\_\mathfrak{R}$ space is $Np\_\mathfrak{R}Np\_\mathfrak{R}$ space.

Proof. Let (Ǵ, $\mathfrak{F}_{\acute{R}p}$ (ℐ))(Ǵ, $\mathfrak{F}_{\acute{R}p}$ (ℐ)) be $Np\_\mathfrak{R}$ $Np\_\mathfrak{R}$ space and (Ḿ, $\mathfrak{F}_{\acute{R}p}$ (Ɏ))(Ḿ, $\mathfrak{F}_{\acute{R}p}$ (Ɏ)) is a subspace on (Ǵ, $\mathfrak{F}_{\acute{R}p}$ (ℐ))(Ǵ, $\mathfrak{F}_{\acute{R}p}$ (ℐ)) and Ḟ be $Np\_\hat{C}\ Np\_\hat{C}$ subset of ḾḾ, v̈ ∉ Ḟv̈ ∉ Ḟ then $\bar{Ḟ}\bar{Ḟ}$ = $\bar{Ḟ}$ ∩ Ḿ $\bar{Ḟ}$ ∩ Ḿ such that $\bar{Ḟ}$ $\bar{Ḟ}$ is $Np\_\hat{C}Np\_\hat{C}$ set in ḾḾ [by definition subspace], so v̈ ∉ $\bar{Ḟ}$v̈ ∉ $\bar{Ḟ}$, v̈ ∈ Ḿ v̈ ∈ Ḿ and v̈∉ $\bar{Ḟ}$*∉ $\bar{Ḟ}$*

Now, $\bar{Ḟ}\bar{Ḟ}$ is $Np\_\hat{C}Np\_\hat{C}$ set in ǴǴ and v̈ ∉ $\bar{Ḟ}$v̈ ∉ $\bar{Ḟ}$ since ǴǴ is $Np - \mathfrak{R}Np - \mathfrak{R}$ space thus ∃ disjoint two $Np\_\hat{O}\ setsNp\_\hat{O}\ sets$ Ǩ, Ĥ Ǩ, Ĥ ∋ ũ ∈ Ǩũ ∈ Ǩ & Ḟ⊆ Ĥ⊆ Ĥ. Furthermore, it Ǩ ∩ Ḿ = K^* Ǩ ∩ Ḿ = K^* and Ĥ ∩ Ḿ = H^* Ĥ ∩ Ḿ = H^* we get H^*,$K^*$$H^*$,$K^*$ are disjoin two $Np\_\hat{O}\ sets$ $Np\_\hat{O}\ sets$ when

$H^* \cup K^*$ = (Ǩ ∩ Ḿ) ∩ (Ĥ ∩ Ḿ) = ∅
$H^* \cup K^*$ = (Ǩ ∩ Ḿ) ∩ (Ĥ ∩ Ḿ) = ∅ and

v̈ ∈ K^*v̈ ∈ K^* because v̈ ∈ Ǩ ,v̈ ∈ Ḿ v̈ ∈ Ǩ ,v̈ ∈ Ḿ, also Ḟ ⊆ H^* ⊆ H^*because $\bar{Ḟ}$ ⊆ Ĥ and $\bar{Ḟ}$ ∩Ḿ =$\bar{Ḟ}$ ⊆ Ĥ and $\bar{Ḟ}$ ∩Ḿ = $\bar{Ḟ}$* $\bar{Ḟ}$* =Ḟ.

Remark (3.24) A space (Ǵ, $\mathfrak{F}_{\acute{R}p}$ (ℐ)) (Ǵ, $\mathfrak{F}_{\acute{R}p}$ (ℐ)) is $NpNp$ T₃-space, only if $Np\_\mathfrak{R}$ $Np\_\mathfrak{R}$ space with also $NpNp$ T₁-space.

Now, proposition that study the Properties of $NpNp$ T₃-space through approximation.

Proposition (3.25) A space (Ǵ, $\mathfrak{F}_{\acute{R}p}$ (ℐ)) (Ǵ, $\mathfrak{F}_{\acute{R}p}$ (ℐ)) is $N\mathfrak{F}_{\acute{R}p}\ N\mathfrak{F}_{\acute{R}p}$, if ($\mathfrak{U}_{\acute{R}p}$ (ℐ)) = Ǵ($\mathfrak{U}_{\acute{R}p}$ (ℐ)) = Ǵ, then ǴǴ is Np Np T₃-space.

Proof. Since $\mathfrak{U}_{\acute{R}p}$ (ℐ)$\mathfrak{U}_{\acute{R}p}$ (ℐ) = ǴǴ, then ǴǴ is discrete space and by using definition (*2.7(2)*) became Ǵ

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Ǵ is $Np\_\mathfrak{R}Np\_\mathfrak{R}$ and $NpNp$ T_1-space, then ǴǴ is $NpNp$ T_3-space.

We study hereditary and $Np-Np-$topological property of $NpNp$ T_3-space.

Proposition (3.26) $Np\ Np$ T_3- space has $Np-Np-$ topological property's

Proof. Suppose $(Ǵ,\mathfrak{F}_{Ŕp}(Ⱥ))(Ǵ,\mathfrak{F}_{Ŕp}(Ⱥ))$ is $Np\ Np$ T_3, Since $NpNp$ T_3- Space is $NpNp$ T_1-Space with $NpNp\_\ \mathfrak{R}\mathfrak{R}$ by definition (2.7(2)) and both are $NpNp--$topological property by remark (2.6(2)) and by Proposition (3.22). Then $NpNp$ T_3-space is topological property.

Proposition (3.27) $NpNp$ T_3-space has hereditary property .

Proof.

Suppose $(Ǵ,\mathfrak{F}_{Ŕp}(Ⱥ))(Ǵ,\mathfrak{F}_{Ŕp}(Ⱥ))$ is $NpNp$ T_3, since $NpNp$ T_3-space is $NpNp$ T_1 and $NpNp\_\ \mathfrak{R}\mathfrak{R}$ by definition (2.7(2))and both are by remark (2.6(2)) and by hrerditary theorem(3.24)then $NpNp$ T_3-space has hereditary property.

Definition (3.28): The space $(Ǵ,\mathfrak{F}_{Ŕp}(Ⱥ))$ $Ǵ,\mathfrak{F}_{Ŕp}(Ⱥ))$ is said to be Strongly Nano Penta Regular space ($SNp\_\mathfrak{R}SNp\_\mathfrak{R}$)space if for each Ḟ is $Np\_\hat{C}\ set\ Np\_\hat{C}\ set$ and for each a point ũũ not belong to Ḟ, ∃Ḟ, ∃ disjoint two $N\_\hat{O}\ set N\_\hat{O}\ set_s$ Ǩ Ǩ and ĤĤ in ǴǴ ∋ ũ∋ ũ ∈ Ĥ ∧∈ Ĥ ∧ Ḟ ⊆ Ǩ Ḟ ⊆ Ǩ. As from the example (3.8).

Proposition (3.29) For space $(Ǵ,\mathfrak{F}_{Ŕp}(Ⱥ))$ $Ǵ,\mathfrak{F}_{Ŕp}(Ⱥ))$ is $N\mathfrak{F}_{Ŕp}$, $N\mathfrak{F}_{Ŕp}$, if ǴǴ is $SNp\_\mathfrak{R}$ $SNp\_\mathfrak{R}$ space then it is $Np\_\mathfrak{R}Np\_\mathfrak{R}$ space. The convers is not true.

Proof. By definition and theorem (2.4) . Then ǴǴ is $Np\_\mathfrak{R}\ Np\_\mathfrak{R}$ space. As shown the example (3.2) is $Np\ Np\_\ \mathfrak{R}\mathfrak{R}$ space but, not $SNpSNp\_\ \mathfrak{R}\mathfrak{R}$ space.

IV. CONCLUSION

The main results of this paper are stated as below:

1. Each $N\_\mathfrak{R}\ N\_\mathfrak{R}$ space is $Np-\mathfrak{R}Np-\mathfrak{R}$ space.
2. A space $(Ǵ,\mathfrak{F}_{Ŕp}(Ⱥ)))(Ǵ,\mathfrak{F}_{Ŕp}(Ⱥ)))$ is $Np-\mathfrak{R}\ Np-\mathfrak{R}$ space only if $\mathfrak{U}_{Ŕp}(Ⱥ)\mathfrak{U}_{Ŕp}(Ⱥ)$ = Ǵ, $\mathbb{L}\mathbb{L}_{Ŕp}$(Ⱥ)=φ and $\mathfrak{U}_{Ŕp}(Ⱥ) \neq \mathbb{L}_{Ŕp}(Ⱥ)\mathfrak{U}_{Ŕp}(Ⱥ) \neq \mathbb{L}_{Ŕp}(Ⱥ)$.
3. A space $(Ǵ,\mathfrak{F}_{Ŕp}(Ⱥ))Ǵ,\mathfrak{F}_{Ŕp}(Ⱥ))$ is $N\mathfrak{F}_{Ŕp}\ N\mathfrak{F}_{Ŕp}$ only if $\mathfrak{U}_{Ŕp}(Ⱥ) \neq Ǵ\ \mathfrak{U}_{Ŕp}(Ⱥ) \neq Ǵ$ and $\mathbb{L}_{Ŕp}(Ⱥ) \neq \varphi\ \mathbb{L}_{Ŕp}(Ⱥ) \neq \varphi$ then $(Ǵ,\mathfrak{F}_{Ŕp}(Ⱥ))Ǵ,\mathfrak{F}_{Ŕp}(Ⱥ))$ is $Np-\mathfrak{R}\ Np-\mathfrak{R}$ space.
4. Every $NpNp$ T_0-space and $Np\_\mathfrak{R}Np\_\mathfrak{R}$ space is $Np\ Np$ T_1- space.
5. Every $Np\_\mathfrak{R}Np\_\mathfrak{R}$ space and $NpNp$ T_1-space is $Np\ Np$ T_2-space.
6. A space $(Ǵ,\mathfrak{F}_{Ŕp}(Ⱥ))\ (Ǵ,\mathfrak{F}_{Ŕp}(Ⱥ))$ is $NpNp$ T_3-space, only if $Np\_\mathfrak{R}Np\_\mathfrak{R}$ space with also $NpNp$ T_1-space
7. Every $SNp\_\mathfrak{R}SNp\_\mathfrak{R}$ space then it is $Np\_\mathfrak{R}Np\_\mathfrak{R}$ space. The convers is not true.

REFERENCES

[1] Bhuvaneswari, K., & Gnanapriya, K. M. (2015). On Nano generalized continuous function in nano topological space. *International Journal of Mathematical Archive*, *6*(6), 182-186.
[2] Murad, N. A., & Jasim, T. H. (2020). *N* Open Sets Via Nano Topological Spaces* (Doctoral dissertation, M. Phil. Thesis, University of Tikrit).
[3] Thivagar, M. L., & Richard, C. (2014). Note on nano topological spaces. *Communicated*, *1*(2.2), 2-3.
[4] Yaseen, R. B., Alobaidi, M. H., & Shihab, A. A. (2022). Applications of Nano Penta (N_P) Separation axioms using N_P_ open sets. *Tikrit Journal of Pure Science*, *27*(5), 117-122.
[5] Yaseen, R. B., Shihab, A. A., & Alobaidi, M. H. (2023, February). On nano Penta topological spaces. In *AIP Conference Proceedings* (Vol. 2414, No. 1). AIP Publishing.