

Measurement of Central Tendencies

Muhajir Sial¹ and Abdullah Abid²

¹Department of Mathematics, Faculty of Education, Sheikh Zaid University, Khost, AFGHANISTAN.

²Department of Mathematics, Faculty of Education, Sheikh Zaid University, Khost, AFGHANISTAN.

Correspondence Author: Muhajir Sial



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ABSTRACT

The main purpose of this article is to understand central tendencies, its application, its types, as well as to learn how to achieve arithmetic mean from grouped and ungrouped data. We will be able to understand geometric and harmonic means and way to calculate them from grouped and ungrouped data. In addition, we will learn which types of mean tends to have more errors and which one to use in which circumstances. We will also be able to know median and mode that are located at the center of data. This article will elucidate the differences between arithmetic mean, median and mode as well as explain ways to calculate mode and median, and pros and cons of median.

Preface: In statistical analysis, we sometimes need to analyze the data with respect to a specific characteristic. This characteristic or number should represent the whole set of data. In statistics, central tendency is a central value for data. Measures of central tendencies are often called averages. The most common measure of central tendency are the arithmetic mean, the median, and the mode. Averages can be divided into two groups. The first type is the average with respect to location and second one is mathematical average. We will explain each one in more detail in this article.

Objectives: The main objective of this article is to understand mean, its types, and usage in daily life.

Methodology: In this article, I have used library research and I have collected data from reliable resources and internet sites.

Keywords- Mean, median, mode, and difference between mean and median.

I. AVERAGE

The term average in mathematics refers to central value that is used in analyzing a set of data. Mean is divided into two types:

1: Arithmetic mean

2 Averages according to location

Arithmetic mean: this type of mean is a simple type of mean and we calculate it by summing up the entries in a set of data divided by the number of entries. Arithmetic mean is shown by the symbol \bar{x} . If we have $x_1, x_2, x_3 \dots x_{n1}$, we calculate the mean of this data by the following formula.

$$\bar{x} = \frac{x_1 + x_2 + x_3 + \dots + x_{n1}}{n}$$

N indicates the number of data.

II. ARITHMETIC MEAN IN GROUPED DATA

If data is divided into groups, we calculate its mean by multiplying class mean with its frequency and divide it by the sum of the frequencies. In a big set of data, calculating arithmetic mean require long mathematical operations. We use the following formula for calculating arithmetic mean.

$$\bar{x} = \frac{\sum_{i=1}^n x_i \cdot f_i}{\sum_{i=1}^n f_i}$$

Alternatively, we can use the following method for calculating mean in grouped data. In this method, we add another column, d, in addition to the column of $x_i \cdot f_i$. This column is called central deviation column. In

d column, we put zero in front of the central class in the table. We then put negative numbers above the zero and positive numbers below zero. We show the mean of the central class by x_0 . We calculate $\frac{\sum_{i=1}^n x_i \cdot f_i}{\sum_{i=1}^n f_i}$ multiplied by C, which is the class length, and then add x_0 . To put this in mathematical form, we write:

$$\bar{x} = x_0 + \left\{ \frac{\sum_{i=1}^n x_i \cdot f_i}{\sum_{i=1}^n f_i} \right\} \cdot C \dots \dots \dots 3$$

In the formula 3, \bar{x} is arithmetic mean, x_0 is class mean, C is class width, and $x_i \cdot f_i$ is the distance of each class from the central class.

III. GEOMETRIC MEAN

When we calculate the geometric mean of two or more numbers, we multiply them with each other and take its nth root. n is the number of entries we have in a set of data. The formula for geometric mean is as follow.

$$GM = \sqrt[n]{x_1 \cdot x_2 \cdot x_3 \dots x_{n1} \dots 4}$$

IV. GEOMETRIC MEAN IN GROUPED DATA

If the data is grouped and we want to calculate its geometric mean, we employ the following formula.

$$GM = \sqrt[N=\sum_{i=1}^n f_i]{x_1^{f_1} \cdot x_2^{f_2} \cdot x_3^{f_3} \dots x_n^{f_n}} = \sqrt[x_1^{f_1} \cdot x_2^{f_2} \cdot x_3^{f_3} \dots x_n^{f_n}]{\frac{1}{N=\sum_{i=1}^n f_i} \dots \dots 5}$$

In formula 4 and 5, $(x_1 \cdot x_2 \cdot x_3 \dots x_{n1})$ are class means, GM geometric mean, and $N = \sum_{i=1}^n f_i$ is the sum of all frequencies. If the data is not grouped and only have frequency, then $x_1 \cdot x_2 \cdot x_3 \dots x_{n1}$ show data. Utilizing logarithm, we can formulate the following formula.

$$\begin{aligned} \log GM &= \log [x_1^{f_1} \cdot x_2^{f_2} \cdot \dots x_n^{f_n}]^{1/N} \Rightarrow \log GM = \frac{1}{N} \log [x_1^{f_1} \cdot x_2^{f_2} \cdot \dots x_n^{f_n}] = \frac{1}{N} [\log(x_1^{f_1}) + \\ \log(x_2^{f_2}) + \dots + \log(x_n^{f_n})] &= \frac{1}{N} [f_1 \log x_1 + f_2 \log x_2 + \dots + f_n \log x_n] = \frac{1}{N} [\sum f_i \log x_i] \Rightarrow GM = \\ &= \text{anti Log} \left[\frac{1}{N} (\sum f_i \log x_i) \right] \\ GM &= \text{anti Log} \left[\frac{1}{N} (\sum f_i \log x_i) \right] \dots \dots \dots (6) \end{aligned}$$

V. HARMONIC MEAN

Harmonic mean is also a type of mean which is used to avoid error in statistical analysis. Its used in analysis of the cost of good produced in an hour or day. This is also useful in finding average velocity, in which velocities differ but distances remain equal. Harmonic mean is the inverse of inverse geometric mean, which

means that we first invert the data and then find the inverse of inverse geometric mean. If $x_1 \cdot x_2 \cdot x_3 \dots x_{n1}$ is the data given, we can calculate its harmonic mean by the following formula:

$$\begin{aligned} Hm &= \frac{1}{\frac{1}{x_1} + \frac{1}{x_2} + \dots + \frac{1}{x_n}} = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \dots + \frac{1}{x_n}} = \frac{n}{\sum_{i=1}^n \frac{1}{x_i}} \\ Hm &= \frac{n}{\sum_{i=1}^n \frac{1}{x_i}} \dots \dots (7) \end{aligned}$$

In formula 7 n indicates the number of entries, x_i each entry, and HM harmonic mean.

VI. HARMONIC MEAN IN GROUPED DATA

If data is grouped or has weight, credit, or frequency, we can calculate its harmonic mean by the following method. If we have $(x_1 \cdot x_2 \cdot x_3 \dots x_{n1})$ is the data given such that $(f_1 \cdot f_2 \cdot f_3 \dots f_n)$ are the frequencies, we can calculate their harmonic mean by the following formula.

$$Hm = \frac{n}{\sum \frac{f_i}{x_i}} \dots \dots (8)$$

In formula 7, n indicates the number of entries, x_i is class mean, f_i is each frequency, and HM is harmonic mean.

VII. MEDIAN

Median is also a type of central tendencies. Median is a quantity calculated by dividing the data in two part, meaning that half of the data remain above the median and half of the data below the median. If data is ungrouped, we organized it increasingly or decreasingly and find its median. If the data is odd, we use the following formula.

$$Md = \frac{x_{\frac{n+1}{2}}}{2} = \frac{n+1}{2} \dots \dots (9)$$

In formula 9, Md is median, x each entry, and n is sum of all entries in a set of data.

Median in grouped data
If the grouped data is organized decreasingly, we use the following formula for calculating median.

$$Md = L_1 + \left(\frac{n}{2} - f \right) \frac{c}{fm} \dots \dots (10)$$

In formula 10, L_1 is the lower boundary of the class where the median is located, n is the sum of frequencies, f is cumulative frequency, and MD is

median. f_m is also frequency that has specific value for each class, and c is the class width.

If, however, the data is organized increasingly, the following formula is used.

$$Md = L_2 - \left(f - \frac{n}{2}\right) \frac{c}{f_m} \dots \dots (11)$$

VIII. MODE

Mode, in a set of data, is the number that has high frequency, or it is a statistical sample that has higher frequency. Mode is widely used in market. For instance, we say that x type of shows is in mode, which means that many people buy and wear them.

8.1 Mode in grouped data

In general, if $(f_1, f_2, f_3 \dots f_n)$ are the frequency of class means, we can calculate its mode by the following formula:

$$Mode = L_1 + \left(\frac{\Delta_1}{\Delta_1 + \Delta_2}\right) \cdot C \dots \dots \dots (16)$$

In the formula 16, L_1 is the lower boundary of the class where mode is located, Δ_1 is the difference

between the frequencies of the class where mode is located and the class before it. Δ_2 is the difference between the frequencies of the mode class and the class that comes after it.

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